Massive modes in Soft Collinear Effective Theory

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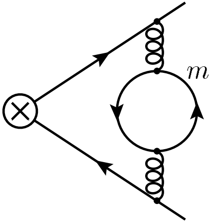
Motivation

- in QCD: massive effects in collider processes via different schemes
  $\rightarrow$ FFNS: $Q \sim m$, kinematics exact, no Log-resummation
  $\rightarrow$ VFNS: $Q \gg m$, massless kinematics, Log-resummation
  $\Rightarrow$ no uniform picture, smooth transitions desirable (relevant e.g. for DIS)
- aim: incorporate massive quarks into jet cross-sections with EFT methods
- in SCET: factorization and resummation for production of primary massive $t\bar{t}$-pairs
  Fleming, Hoang, Mantry, Stewart (2008)
  $\rightarrow$ still missing: systematic treatment of secondary massive quarks



Outline

1. SCET
2. Factorization theorems for event shape variables
3. Mass modes
   - Setup
   - $e^+e^- \to jets$
4. Summary & Outlook

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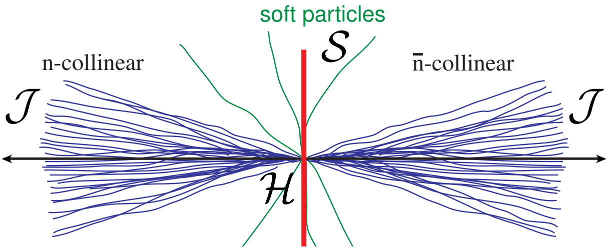
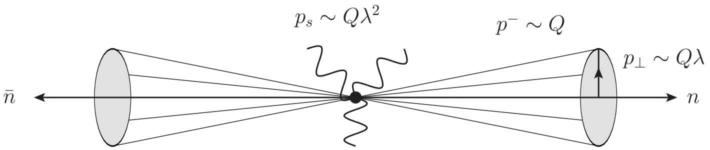
What is SCET?

- SCET= Soft-collinear effective theory
  `Bauer, Fleming, Luke (2000), Bauer et al. (2001)`
- EFT of QCD for energetic hadrons & jets describing the interactions of soft and collinear particles
  $\to$ B-decays ($B \to X_s\gamma$, $B \to \pi l \nu$, ...)
  $\to$ hard scattering, jets ($e^- p \to e^- X$, $pp \to X l^+ l^-$, $e^+ e^- \to jets$, ...)
- allows to factorize short distance physics from long range physics:
  $d\sigma = \mathcal{H} \cdot \mathcal{J} \otimes \mathcal{S}$

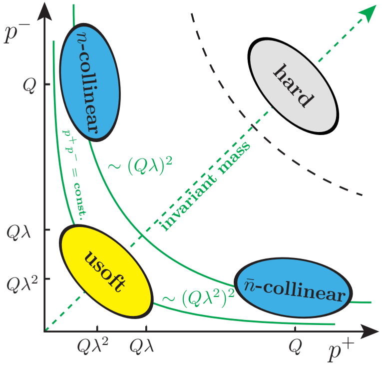


Scales in jets



- small fluctuations around mass-shell for boosted particles
  $\rightarrow$ lightcone coordinates $n^\mu = (1, 0, 0, 1)$, $\bar{n}^\mu = (1, 0, 0, -1)$
  $\rightarrow p^\mu = \bar{n} \cdot p \, \frac{n^\mu}{2} + n \cdot p \, \frac{\bar{n}^\mu}{2} + p_\perp \equiv (p^-, p^+, p_\perp)$
  $\rightarrow p^2 = p^- p^+ + p_\perp^2$
- for n-collimated jets with energy $Q^2 \gg m_{jet}^2 \gg \Lambda_{QCD}^2$: $p^- \gg p_\perp \gg p^+$
  $\rightarrow$ language of effective field theory appropriate
  $\rightarrow p^- \sim Q$, $p_\perp \sim Q\lambda$, $p^+ \sim Q\lambda^2$ for collinear modes
  with power counting parameter $\lambda = m_{jet}/Q$
  typically $\lambda = \sqrt{\Lambda_{QCD}/Q}$

Degrees of freedom (SCET I)



mode	$p^\mu = (+,-,\perp)$	p^2	fields
hard	$Q(1,1,1)$	Q^2	$-$
n-collinear	$Q(\lambda^2,1,\lambda)$	$Q^2\lambda^2$	ξ_n, A_n^μ
$\bar{n}$-collinear	$Q(1,\lambda^2,\lambda)$	$Q^2\lambda^2$	$\xi_{\bar{n}}$, $A_{\bar{n}}^\mu$
ultrasoft	$Q(\lambda^2,\lambda^2,\lambda^2)$	$Q^2\lambda^4$	q_{us}, A_{us}^μ

Benefits of EFT treatment

- definite power counting
  → simplified expanded calculation from the start with appropriate degrees of freedom
  → systematic treatment of higher order effects
- emergence of symmetries for kinematical regime
  in SCET: gauge symmetry for each sector
  → factorization (separation of kinematical regions)

  $d\sigma = \mathcal{H} \cdot \mathcal{J} \otimes \mathcal{S}$ (to all orders in α_s, LO in λ)
  $\mathcal{H} \rightarrow$ hard sector (matching coefficient)
  $\mathcal{J} \rightarrow$ collinear sector (jet function)
  $\mathcal{S} \rightarrow$ ultrasoft sector (soft function)

  → operator based definitions
  → systematic incorporation of power corrections possible
- resummation of large logarithms via RG evolution

Resummation: SCET counting

- in QCD: IR soft + collinear divergences $\leftrightarrow$ in SCET: UV divergences
  $\rightarrow$ in DIMREG $1/\epsilon^2$-divergences at one-loop
  $\leftrightarrow$ Sudakov double log's, e.g. $\log^2 \frac{\mu_H}{\mu_J}$, $\log^2 \frac{\mu_J}{\mu_S} \sim \log^2 \lambda \gg 1$
- resummation of series $1 + \alpha_s \log^2 \lambda + \alpha_s^2 \log^4 \lambda + \ldots$ at LO required $\rightarrow$ LL
- performed with renormalization group equations
  $\rightarrow$ each sector evaluated at its natural scale,
  Log-resummation in evolution factors U_H, U_J, U_S
- table for resummation of logarithms:

series		one-loop	two-loops	three-loops	four-loop
LL	$\alpha_s^n \ln^{n+1}$	$1/\epsilon^2$	–	–	–
NLL	$\alpha_s^n \ln^n$	$1/\epsilon$	$1/\epsilon^2$	–	–
NNLL	$\alpha_s^n \ln^{n-1}$	ϵ^0	$1/\epsilon$	$1/\epsilon^2$	–
N^3LL	$\alpha_s^n \ln^{n-2}$	ϵ	ϵ^0	$1/\epsilon$	$1/\epsilon^2$

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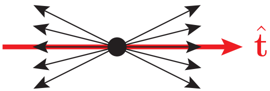
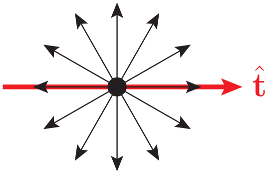
Event shapes: Thrust

- event shape variables: describe geometrically kinematics of final state for an inclusive event
- examples: thrust, jet mass, spherocity, jet broadening, ...
- thrust: $\tau \equiv 1 - \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \in [0, \frac{1}{2}]$



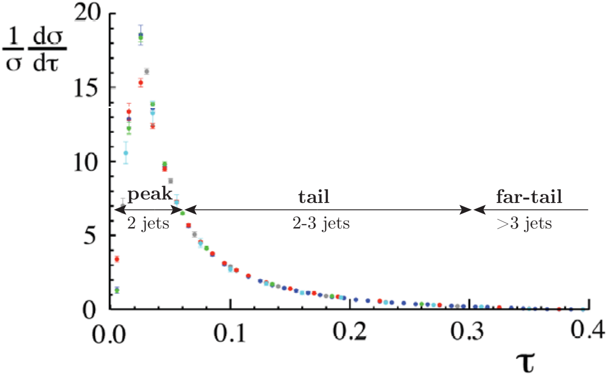
back-to-back: $\tau \to 0$



isotropic: $\tau \to \frac{1}{2}$

Thrust distribution

- thrust distribution from LEP data ($e^+e^- \to jets$)



- peak region ($\tau \sim \Lambda_{QCD}/Q$): expansion parameter $\lambda = \sqrt{\Lambda_{QCD}/Q}$
- tail region ($\tau \gg \Lambda_{QCD}/Q$): expansion parameter $\lambda = \sqrt{\tau}$

Factorization theorem for thrust

`Fleming, Hoang, Mantry, Stewart (2007)`
`Bauer, Fleming, Lee, Sterman (2008)`

SCET result for $\tau \ll 1$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} = Q^2 \sigma_0 H_Q(Q,\mu) \int d\ell \, J_\tau(Q\ell,\mu) S_\tau \left(Q\tau - \ell, \mu\right)$$

$Q \to$ CM energy, $\sigma_0 \to$ cross-section at LO

Ingredients:
$H_Q(Q,\mu)$ (hard function), $J_\tau(s,\mu)$ (jet function), $S_\tau\,(l,\mu)$ (soft function)

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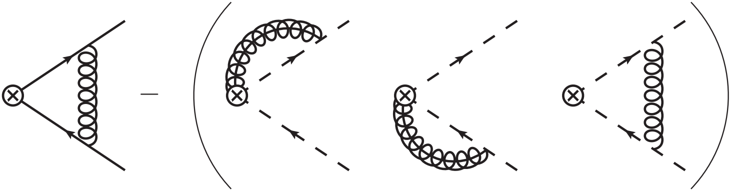
hard function: $H_Q(Q,\mu) = |C_V(Q,\mu)|^2$
$\rightarrow C_V(Q,\mu)$: matching coefficient QCD - SCET at scale $\mu_H \sim Q$



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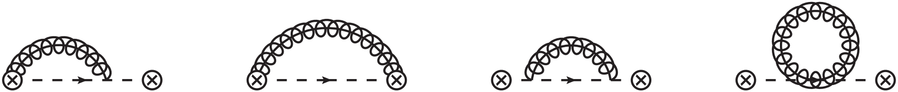
thrust jet function: $J_\tau(s,\mu) = \int ds'\; J_n(s',\mu) J_{\bar{n}}(s-s',\mu)$

$\rightarrow J_n, J_{\bar{n}}$: jet rate in terms of its invariant mass s,
cuts through forward scattering amplitude



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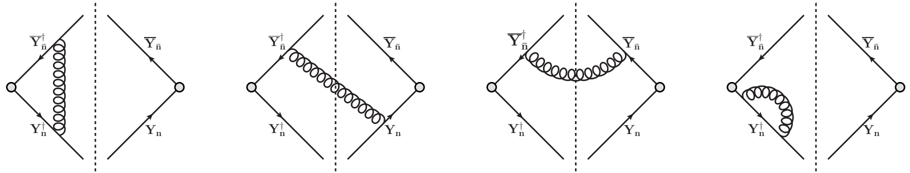
thrust soft function: $S_\tau(l,\mu) \equiv \int dk_R \, dk_L \, \delta(l - k_R - k_L) \, S_{\mathrm{h}}(k_R, k_L, \mu)$

$\rightarrow$ S_{h}: describes all soft particles in each hemisphere (L/R)

$\rightarrow$ $\mu_S \sim Q\lambda^2 \sim \Lambda_{QCD}$: $S_\tau = S_\tau^m$: non-perturbative model

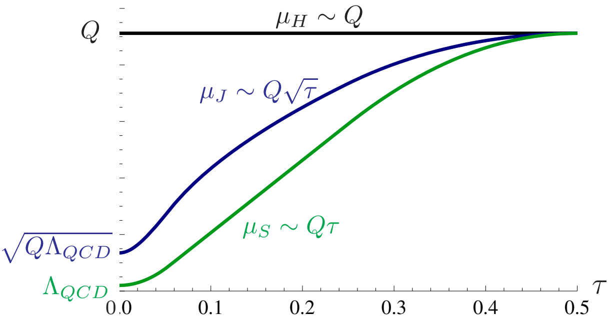
$\rightarrow$ $\mu_S \sim Q\lambda^2 \gg \Lambda_{QCD}$: $S_\tau = S_\tau^p \otimes S_\tau^M$: S_τ^p partonic piece (perturbative)



Profile functions

Profile functions: Parametrization of renormalization scales in terms of thrust
$\rightarrow$ continuous transition between peak, tail and far tail region

region		μ_H	μ_J	μ_S
peak	$\tau \sim \Lambda_{\mathrm{QCD}}/Q$	$\sim Q$	$\sim \sqrt{Q\Lambda_{\mathrm{QCD}}}$	$\sim \Lambda_{\mathrm{QCD}}$
tail	$\Lambda_{\mathrm{QCD}}/Q \ll \tau \leq 1/3$	$\sim Q$	$\sim Q\sqrt{\tau}$	$\sim Q\tau$
far-tail	$1/3 \leq \tau \leq 1/2$	$\sim Q$	$\sim Q$	$\sim Q$



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Mass modes

introduce particle species with mass m

$\rightarrow$ additional scaling parameter: $\lambda_m = m/Q$

$\rightarrow$ new degrees of freedom: "mass modes"

$\Rightarrow$ How to include them into massless setup?
Factorization theorem? RG evolution?

Goal: Construct a hierarchy of EFTs depending on $\lambda_m \leftrightarrow \lambda$

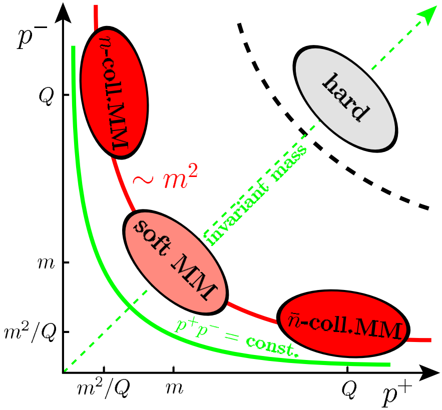
$\rightarrow$ I. $\lambda_m > 1 > \lambda > \lambda^2$

$\rightarrow$ II. $1 > \lambda_m > \lambda > \lambda^2$

$\rightarrow$ III. $1 > \lambda > \lambda_m > \lambda^2$

$\rightarrow$ IV. $1 > \lambda > \lambda^2 > \lambda_m$

Degrees of freedom



- n-collinear mass modes: $p^\mu \sim Q(1, \lambda_m^2, \lambda_m)$
- $\bar{n}$-collinear mass modes: $p^\mu \sim Q(\lambda_m^2, 1, \lambda_m)$
- soft mass modes: $p^\mu \sim Q(\lambda_m, \lambda_m, \lambda_m)$

Scaling situations

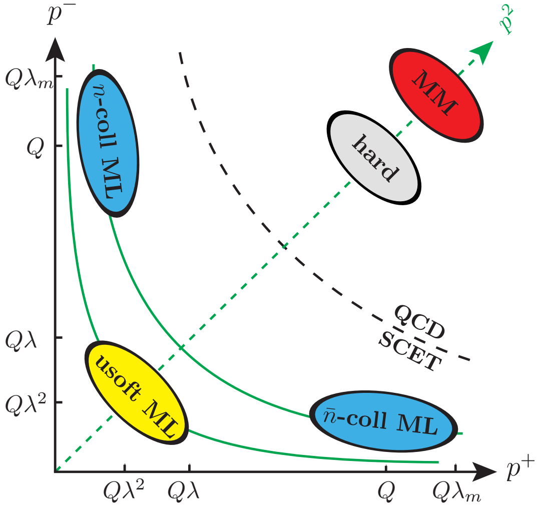


Figure: I. $\lambda_m > 1 > \lambda > \lambda^2$

Scaling situations

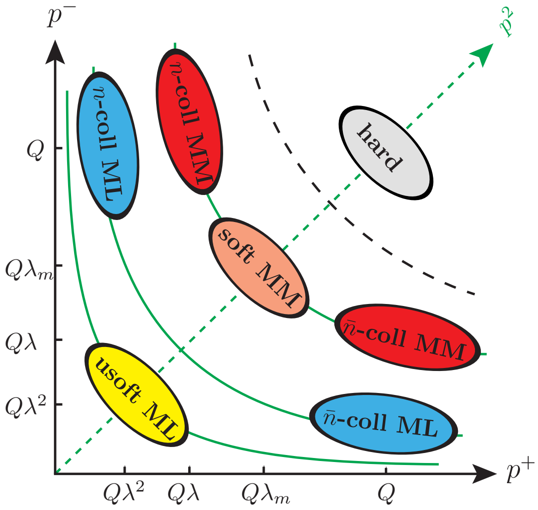


Figure: II. $1 > \lambda_m > \lambda > \lambda^2$

Scaling situations

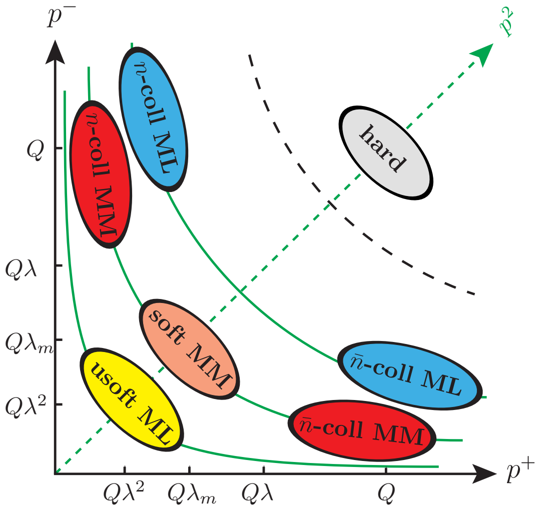


Figure: III. $1 > \lambda > \lambda_m > \lambda^2$

Scaling situations

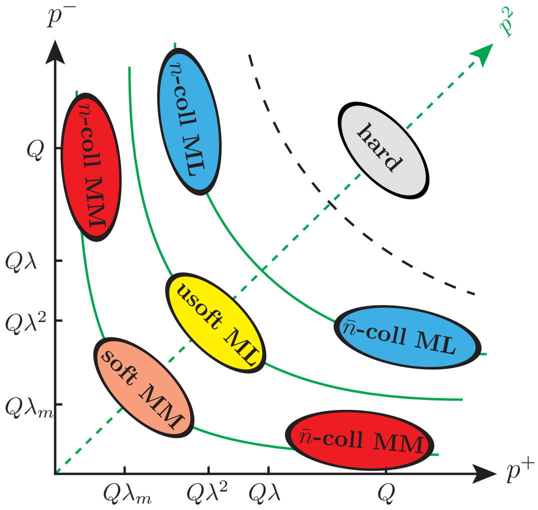
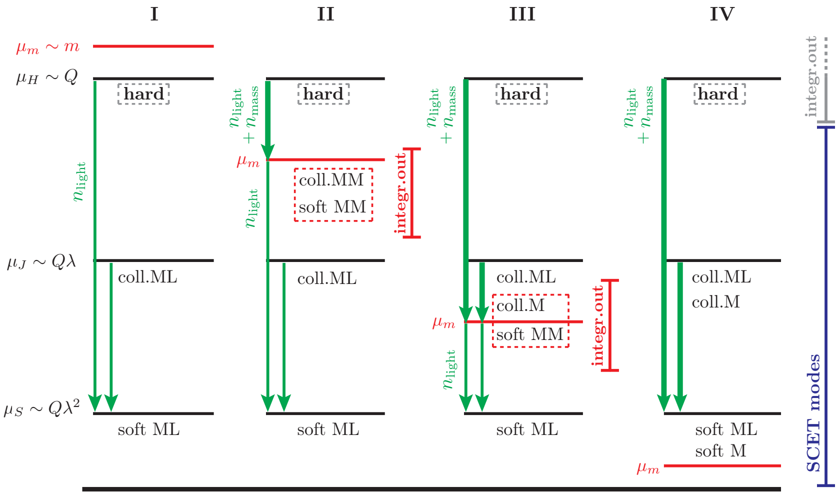


Figure: IV. $1 > \lambda > \lambda^2 > \lambda_m$

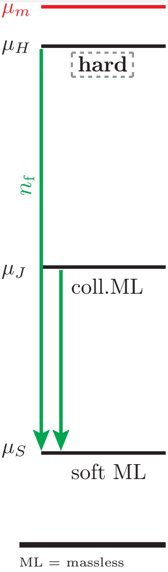
Summary: Mass mode setup

Top-down evolution: evolve to $\mu \sim \mu_S$



MM = mass-mode, ML = massless, M = massive

Scenario I: $\lambda_m > 1 > \lambda > \lambda^2$



integrate out mass modes at QCD level
$\rightarrow$ modification of hard matching coefficient,
otherwise like in massless SCET

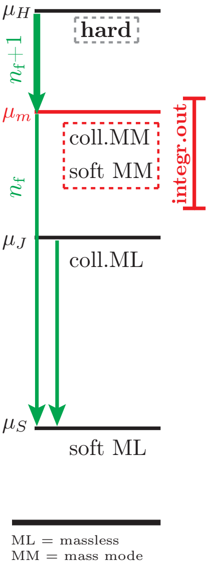
$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^I(\mu_H)\right|^2 U_{H_Q}^{(n_f)}(\mu_H, \mu_S) \int d\ell \int ds \\ \times J_{0,\tau}(s, \mu_J) U_J^{(n_f)}(Q\ell - s, \mu_S, \mu_J) S_{0,\tau}(Q\tau - \ell, \mu_S)$$

$U_{H_Q}^{(n_f)}$, $U_J^{(n_f)}$: evolution factors (n_f light flavours)

$$\mathcal{C}^I(\mu_H) = C_0(\mu_H) + \delta F_m^{\mathrm{QCD}}$$

C_0: massless matching coefficient
$\delta F_m^{\mathrm{QCD}}$: massive QCD contribution (OS scheme)
$\rightarrow$ decoupling for $m/Q \rightarrow \infty$

Scenario II : $1 \gg \lambda_m \gg \lambda \gg \lambda^2$



mass modes enter SCET, but integrated out before the jet scale

$\rightarrow$ modification of the matching coefficient at μ_H

$\rightarrow$ additional matching contribution at μ_m

$\rightarrow$ pay attention: α_s-decoupling

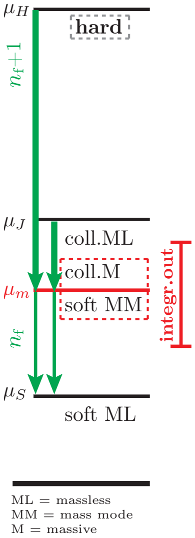
$\rightarrow$ massless jet & soft function

$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_{H_Q}^{(n_f+1)}(\mu_H, \mu_m) \left|\mathcal{M}_{H_Q}(\mu_m)\right|^2 U_{H_Q}^{(n_f)}(\mu_m, \mu_S)$$
$$\times \int d\ell \int ds\, J_{0,\tau}(s, \mu_J) U_J^{(n_f)}(Q\ell - s, \mu_S, \mu_J) S_{0,\tau}(Q\tau - \ell, \mu_S)$$

$$\mathcal{C}^{II}(\mu_H) = \mathcal{C}^{(I)}(\mu_H) - \delta F_m^{\text{SCET}}(\mu_H) \ (+\ \text{OS} \leftrightarrow \overline{MS})$$
$$\mathcal{M}_{H_Q}(\mu_m) = 1 + \delta F_m^{\text{SCET}}(\mu_m) \ (+\ \alpha_s\text{-decoupling})$$

δF_m^{SCET}: massive SCET contribution ($\overline{MS}$ scheme)

Scenario III: $1 \gg \lambda \gg \lambda_m \gg \lambda^2$



massive and massless collinear modes fluctuate over comparable scales ($\lambda_m \leq \lambda$)

$\rightarrow$ assign collinear massless scaling (keep $m \neq 0$)

$\rightarrow$ modification of the jet function at μ_J

$\rightarrow$ additional jet matching contribution at μ_m

$\rightarrow$ massless soft function

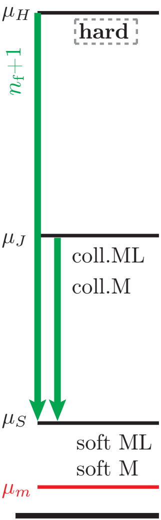
$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{II}(\mu_H)\right|^2 U_{H_Q}^{(n_f+1)}(\mu_H,\mu_m)\left|\mathcal{M}_{H_Q}(\mu_m)\right|^2 U_{H_Q}^{(n_f)}(\mu_m,\mu_S)$$
$$\times \int d\ell \int ds \int ds' \int ds'' J_{0+m,\tau}(s,\mu_J) U_J^{(n_f+1)}(s'-s,\mu_m,\mu_J)$$
$$\times \mathcal{M}_J(s''-s',\mu_m) U_J^{(n_f)}(s''-Q\ell,\mu_S,\mu_m) S_{0,\tau}(Q\tau-\ell,\mu_S)$$

$$J_{0+m,\tau}(s,\mu_J) = J_{0,\tau}(s,\mu_J) + \delta\hat{J}_{\mathrm{m},\tau}(s,\mu_J) + \theta(s-4m^2)\mathcal{R}_J(s)$$
$$\mathcal{M}_J(s,\mu_m) = \mu_m^2 \delta\left(s/\mu_m^2\right) - \delta\hat{J}_{\mathrm{m},\tau}(s,\mu_m) \ (+\ \alpha_s\text{-decoupling})$$

$\delta\hat{J}_{\mathrm{m},\tau}$: distributive piece of jet function

$\mathcal{R}_J$: real radiation piece of jet function

Scenario IV: $1 \gg \lambda \gg \lambda^2 \gg \lambda_m$



ML = massless
M = massive

massive soft and massless usoft modes fluctuate over comparable scales ($\lambda_m \leq \lambda^2$)

$\rightarrow$ assign usoft massless scaling (keep $m \neq 0$)!

$\rightarrow$ all structures get massive contributions

$\rightarrow$ massive modes stay in the game to the end

$$\frac{d\sigma}{d\tau} \sim \left|\mathcal{C}^{ll}(\mu_H)\right|^2 U_{HQ}^{(n_f+1)}(\mu_H,\mu)\int d\ell \int ds' \times J_{0+m,\tau}(s,\mu_J)U_J^{(n_f+1)}(Q\ell - s,\mu_S,\mu_J)S_{0+m,\tau}(Q\tau-\ell,\mu_S)$$

$$S_{0+m,\tau}(\ell,\mu) = S_{0,\tau}(l,\mu) + \delta\hat{S}_{\mathrm{m},\tau}(l,\mu) + \theta(l-2m)\mathcal{R}_S(l)$$

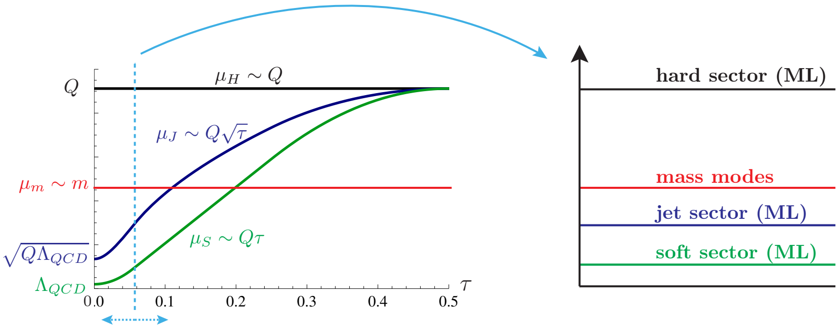
$\delta\hat{S}_{\mathrm{m},\tau}$: distributive piece of massive soft function

$\mathcal{R}_S$: real radiation piece of massive soft function

Scenarios in thrust distribution

Several scenarios in one single event shape spectrum (for $Q > m$)

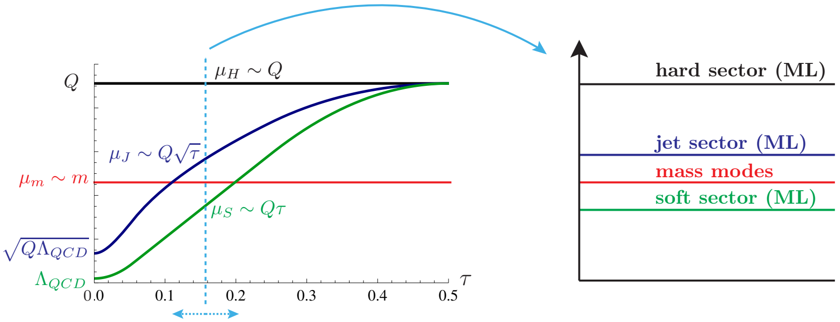
Scenario II:



Scenarios in thrust distribution

Several scenarios in one single event shape spectrum (for $Q > m$)

Scenario III:



Scenarios in thrust distribution

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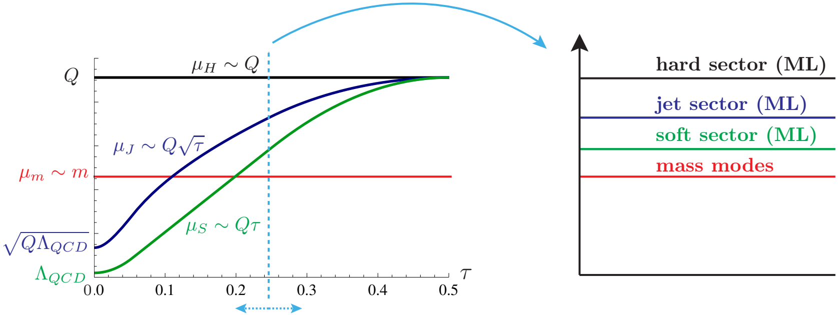
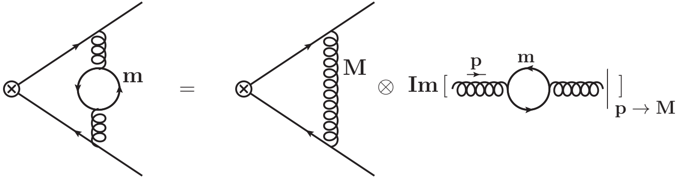
Scenario IV:



Calculation

- calculation of secondary massive quark contributions at two loop
- dispersion integration: amplitudes for massive gauge boson, then convolution with vacuum polarization



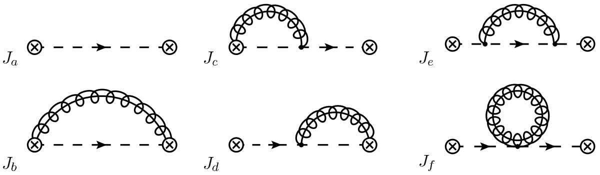
$$\Pi^{\text{eff}}_{\mu\nu}(q^2) = \frac{\alpha_s}{\pi}\int_{4m^2}^{\infty} dM^2 \frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)}{q^2 - M^2 + i\epsilon}\mathcal{V}(M^2, m^2)$$

- remark: be aware of double counting → soft-bin subtractions!

Calculation

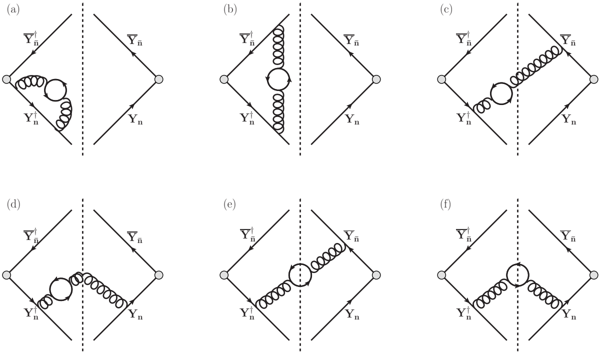
- Wilson coefficient analytically

- jet function analytically



Calculation

- soft function: complicated phase space!
  $\to$ does not fully allow for computation with dispersive technique
  $\to$ partially numerically



- so far no non-perturbative physics implemented

Consistency checks

- anomalous dimensions mass-independent (same result for M and ML)
- continuous transition between different scenarios up to $\mathcal{O}(\alpha_s^3)$

$$\frac{d\sigma}{d\tau}\bigg|_{\mu_m \overset{>}{\to} \mu_H} = \frac{d\sigma}{d\tau}\bigg|_{\mu_m \overset{<}{\to} \mu_H}$$

$$\frac{d\sigma}{d\tau}\bigg|_{\mu_m \overset{>}{\to} \mu_J} = \frac{d\sigma}{d\tau}\bigg|_{\mu_m \overset{<}{\to} \mu_J}$$

$$\frac{d\sigma}{d\tau}\bigg|_{\mu_m \overset{>}{\to} \mu_S} = \frac{d\sigma}{d\tau}\bigg|_{\mu_m \overset{<}{\to} \mu_S}$$

- massless limit reached

$$\mathcal{C}^{II}(Q, m, \mu_H) \overset{m\to 0}{\longrightarrow} C_0(Q, \mu_H)$$

$$J_{0+m,\tau}(s, m, \mu_J) \overset{m\to 0}{\longrightarrow} J_{0,\tau}(s, \mu_J)$$

$$S_{0+m,\tau}(\ell, m, \mu) \overset{m\to 0}{\longrightarrow} S_{0,\tau}(s, \mu_J)$$

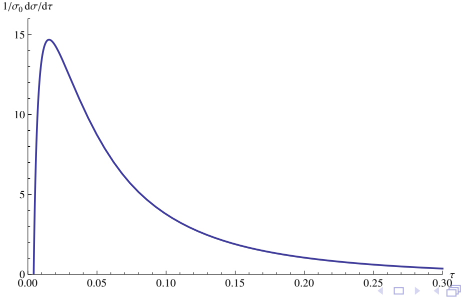
- scenario IV: fixed order gives expanded QCD result ($\mu_H = \mu_J = \mu_S = \mu_m$):

$$\frac{d\sigma}{d\tau}\bigg|_{\mathrm{FO}} \sim \left\{1 + 2\mathrm{Re}\left[\delta F_m^{\mathrm{QCD}}\right]\right\}\delta(\tau) - \theta(Q^2\tau - 4m^2)\mathcal{R}_J(Q^2\tau) - \theta(Q\tau - 2m)\mathcal{R}_S(Q\tau)$$

Plots for $Q = 14{,}35$ GeV

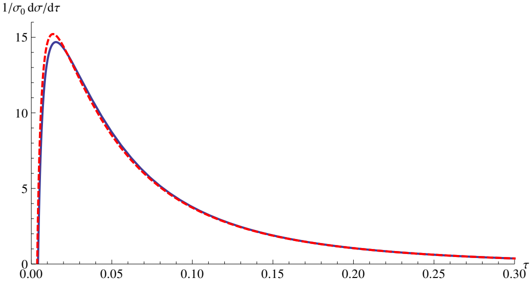
- comparison of ML (5 light flavours) and M (4 light flavours + massive b) thrust distribution
  preliminary, so far no non-perturbative power corrections!
- $Q = 14, 35$ GeV $\leftrightarrow$ data from PETRA
- determination of $\alpha_s(M_z)$: $Q = 35...207$ GeV
  `Abbate, Fickinger, Hoang, Mateu, Stewart (2011, 2012)`
- our default value for coupling constant: $\alpha_s(M_z) = 0.118$

Thrust distribution: massive ($Q = 35$ GeV)

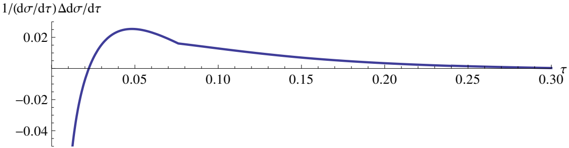


Plots for $Q = 35$ GeV

Thrust distribution: massive (blue, continous) vs. massless (red, dashed)

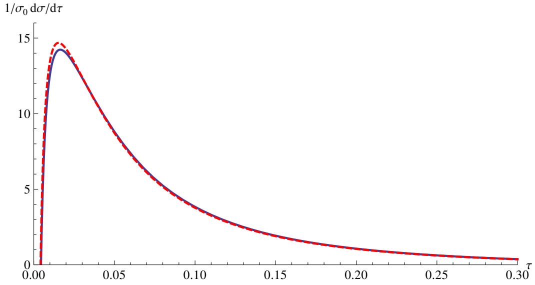


relative deviation massive - massless

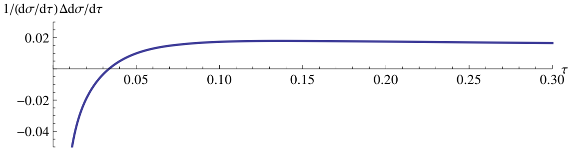


Plots for $Q = 35$ GeV

Thrust distribution: $\alpha_s(M_z) = 0.119$ (blue, continous) vs. $\alpha_s(M_z) = 0.118$ (red, dashed)



relative deviation $\alpha_s(M_z) = 0.119$ - $\alpha_s(M_z) = 0.118$



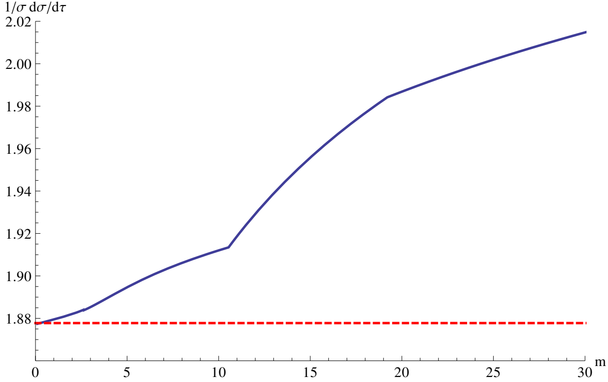
Plots for $Q = 35$ GeV

Comparison of ML (5 light flavours) and M (4 light flavours + massive *b*) thrust distribution

preliminary, so far no non-perturbative power corrections!

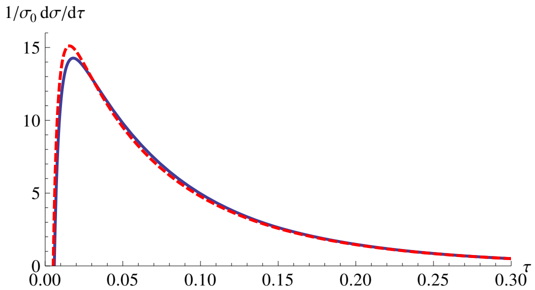
$Q = 35$ GeV $\leftrightarrow$ data from PETRA

Thrust distribution: fixed $\tau = 0.15$, vary mass m (massless: red, dashed)

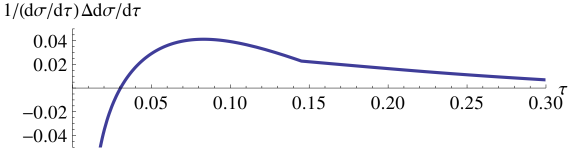


Plots for $Q = 14$ GeV

Thrust distribution: massive (blue, continous) vs. massless (red, dashed)



relative deviation massive-massless



Outline

1 SCET

2 Factorization theorems for event shape variables

3 Mass modes
- Setup
- $e^+e^- \to jets$

4 Summary & Outlook

Summary & Outlook

- SCET appropriate tool for jet physics
- inclusion of heavy quark masses necessary for precision collider physics
- setup for secondary massive quarks in terms of mass modes
- calculation of all ingredients for thrust distribution at N^3LL
- possible applications:
  1. bottom mass effects in α_s determination at N^3LL
  2. heavy quark effects in data from B factories
  3. massive effects for parton distribution functions in deep inelastic scattering
  4. massive gauge boson/Higgs/gluino production
  5. . . .

Summary & Outlook

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  5. ...

Thank you!

Outline

5 Backup-slides

Lagrangian

- label formalism as in HQET and vNRQCD:
  collinear quark momentum $\rightarrow$ label momentum + residual momentum

$$p^\mu = \tilde{p}^\mu + k^\mu, \text{ with } \tilde{p}^\mu = Q(0,1,\lambda),\ k^\mu = Q(\lambda^2,\lambda^2,\lambda^2)$$

- field decomposition

$$\psi(x) \rightarrow \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x}\left(\xi_{n,\tilde{p}}(x) + \xi_{\bar{n},\tilde{p}}(x)\right)$$

- Lagrangian at LO in λ from $\mathcal{L}_{\mathrm{QCD}}$ $(m=0)$

$$\begin{aligned} \mathcal{L}_{n,\tilde{p}} &= \bar{\xi}_{n,\tilde{p}}(in\cdot D + \frac{p_\perp^2}{\bar{n}\cdot p})\frac{\bar{\not n}}{2}\xi_{n,\tilde{p}} \\ &+\bar{\xi}_{n,\tilde{p}+\tilde{q}}\left[gn\cdot A_{n,\tilde{q}} + g\not{A}^\perp_{n,\tilde{q}}\frac{\not{p}_\perp}{\bar{n}\cdot p} + \frac{\not{p}_\perp + \not{q}_\perp}{\bar{n}\cdot(p+q)}g\not{A}^\perp_{n,\tilde{q}}\right. \\ &\left. -\frac{\not{p}_\perp + \not{q}_\perp}{\bar{n}\cdot(p+q)}g\bar{n}\cdot A_{n,\tilde{q}}\frac{\not{p}_\perp}{\bar{n}\cdot p}\right]\frac{\bar{\not n}}{2}\xi_{n,\tilde{p}} + \mathcal{O}(\alpha_s) + \mathcal{O}(\lambda) \end{aligned}$$

- $iD^\mu = i\partial^\mu + gA^\mu_{us}$
- ultrasoft modes: $\mathcal{L}_{\mathrm{us}} = \mathcal{L}_{\mathrm{QCD}}(\psi_{\mathrm{us}}, A_{\mathrm{us}})$

Lagrangian

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\mathcal{L}_{n,\tilde{p}} &= \bar{\xi}_{n,\tilde{p}}\left(in\cdot D + \frac{p_\perp^2}{\bar{n}\cdot p}\right)\frac{\bar{\not{n}}}{2}\xi_{n,\tilde{p}} \\
&\quad + \bar{\xi}_{n,\tilde{p}+\tilde{q}}\Bigg[gn\cdot A_{n,\tilde{q}} + g\not{A}^\perp_{n,\tilde{q}}\frac{\not{p}_\perp}{\bar{n}\cdot p} + \frac{\not{p}_\perp+\not{q}_\perp}{\bar{n}\cdot(p+q)}g\not{A}^\perp_{n,\tilde{q}} \\
&\quad - \frac{\not{p}_\perp+\not{q}_\perp}{\bar{n}\cdot(p+q)}g\bar{n}\cdot A_{n,\tilde{q}}\frac{\not{p}_\perp}{\bar{n}\cdot p}\Bigg]\frac{\bar{\not{n}}}{2}\xi_{n,\tilde{p}} + \mathcal{O}(\alpha_s) + \mathcal{O}(\lambda)
\end{aligned}$$

  labels conserved by usoft interactions

- $iD^\mu = i\partial^\mu + gA^\mu_{us}$
- ultrasoft modes: $\mathcal{L}_{\rm us} = \mathcal{L}_{\rm QCD}(\psi_{\rm us}, A_{\rm us})$

Lagrangian

- label formalism as in HQET and vNRQCD:
  collinear quark momentum → label momentum + residual momentum

$$p^\mu = \tilde{p}^\mu + k^\mu, \text{ with } \tilde{p}^\mu = Q(0,1,\lambda),\ k^\mu = Q(\lambda^2,\lambda^2,\lambda^2)$$

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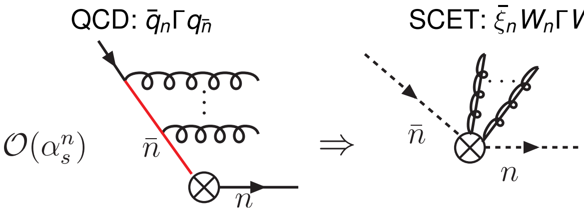
- Lagrangian at LO in λ from $\mathcal{L}_{\rm QCD}$ $(m=0)$

$$\begin{aligned}\mathcal{L}_{n,\tilde{p}} \quad = \quad & \bar{\xi}_{n,\tilde{p}}(in\cdot D + \frac{p_\perp^2}{\bar{n}\cdot p})\frac{\bar{\not{n}}}{2}\xi_{n,\tilde{p}} \\ & + \bar{\xi}_{n,\tilde{p}+\tilde{q}}\left[gn\cdot A_{n,\tilde{q}} + g\not{A}^\perp_{n,\tilde{q}}\frac{\not{p}_\perp}{\bar{n}\cdot p} + \frac{\not{p}_\perp + \not{q}_\perp}{\bar{n}\cdot(p+q)}g\not{A}^\perp_{n,\tilde{q}}\right. \\ & \left. - \frac{\not{p}_\perp + \not{q}_\perp}{\bar{n}\cdot(p+q)}g\bar{n}\cdot A_{n,\tilde{q}}\frac{\not{p}_\perp}{\bar{n}\cdot p}\right]\frac{\bar{\not{n}}}{2}\xi_{n,\tilde{p}} + \mathcal{O}(\alpha_s) + \mathcal{O}(\lambda)\end{aligned}$$

labels affected by collinear interactions

- $iD^\mu = i\partial^\mu + gA^\mu_{us}$
- ultrasoft modes: $\mathcal{L}_{\rm us} = \mathcal{L}_{\rm QCD}(\psi_{\rm us}, A_{\rm us})$

Wilson lines



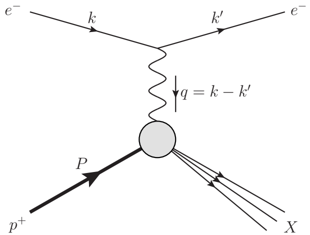
$$W_n = \sum_{k=0}^{\infty} \sum_{\text{perm}} \frac{(-g)^k}{k!} \left(\frac{\bar{n} \cdot A_n^{(k)} \dots \bar{n} \cdot A_n^{(1)}}{\bar{n} \cdot q_1 \dots \bar{n} \cdot \sum_{i=1}^{k} q_i} \right)$$

- integrate out off-shell particles
- emission of several collinear gluons in interaction vertex (not λ-suppressed)
- usoft-collinear decoupling (field redefinition): $\xi_n \to Y_n \xi_n^{(0)}$

$$Y_n = \sum_{k=0}^{\infty} \sum_{\text{perm}} \frac{(-g)^k}{k!} \left(\frac{n \cdot A_{us}^{(k)} \dots n \cdot A_{us}^{(1)}}{n \cdot q_1 \dots n \cdot \sum_{i=1}^{k} q_i} \right)$$

- Wilson lines guarantee gauge invariance of currents

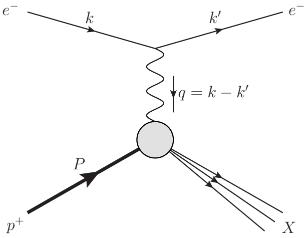
Deep inelastic scattering



- effect of heavy quarks (c, b) on light quark PDFs for differential cross-sections?
- in standard QCD: schemes interpolating between fixed and variable flavours

$\Rightarrow$ apply mass mode setup to DIS (crossed process to $e^+e^- \to$ *jets*)

Deep inelastic scattering

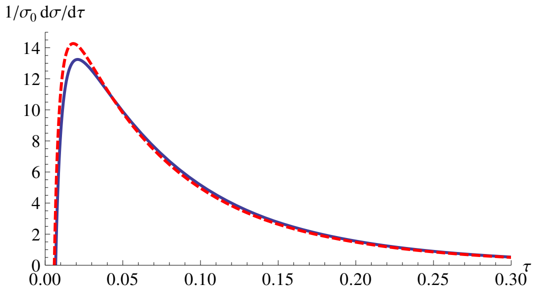


kinematical regimes:

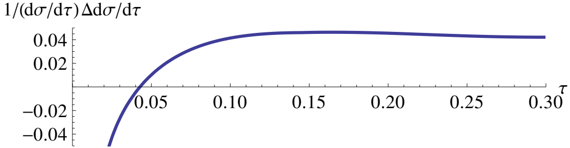
- characterized by Bjorken variable $x \equiv -\frac{q^2}{2P \cdot q}$
- $1 - x \gg \Lambda_{QCD}/Q$: OPE regime
  $\rightarrow$ hard matching & pdf's
- $1 - x \sim \Lambda_{QCD}/Q$: endpoint region
  $\rightarrow$ additionally: integrate out final state masses $\sim Q^2(1-x)$
  $\rightarrow x \rightarrow 1$ in DIS $\sim \tau \rightarrow 0$ in $e^+e^- \rightarrow$ *jets*

Plots for $Q = 14$ GeV

Thrust distribution: $\alpha_s(M_z) = 0.120$ (blue, continous) vs. $\alpha_s(M_z) = 0.118$ (red, dashed)



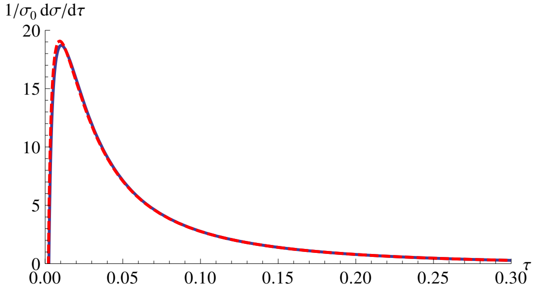
relative deviation $\alpha_s(M_z) = 0.120$ - $\alpha_s(M_z) = 0.118$



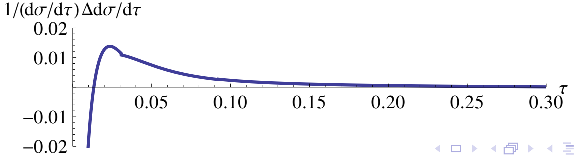
Plots for $Q = 91$ GeV

$Q = 91.2$ GeV $\leftrightarrow$ data from SLC, LEP

Thrust distribution: massive (blue, continous) vs. massless (red, dashed)

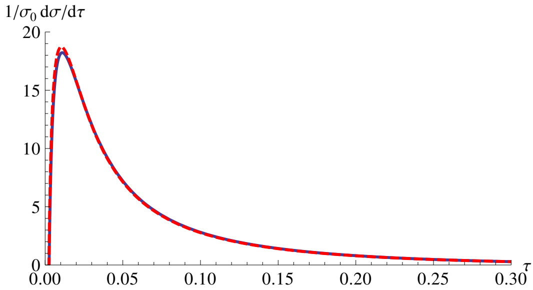


relative deviation massive-massless



Plots for $Q = 91$ GeV

Thrust distribution: $\alpha_s(M_z) = 0.119$ (blue, continous) vs. $\alpha_s(M_z) = 0.118$ (red, dashed)



relative deviation $\alpha_s(M_z) = 0.119$ - $\alpha_s(M_z) = 0.118$