

Introduction into Standard Model and Precision Physics

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Overview

- 1 Electroweak phenomenology before the GSW model**
- 2 The principle of local gauge invariance**
- 3 The Standard Model of electroweak interaction**
- 4 Electroweak phenomenology**
- 5 Quantum field theories and higher perturbative orders**
- 6 Electroweak Standard Model — radiative corrections**



1 Electroweak phenomenology before the GSW model

Some phenomenological facts:

- **discovery of the weak interaction via radioactive β -decay of nuclei:**



- **terminology “weak”:** interaction at low energy has very short range
 \hookrightarrow long life time of weakly decaying particles:

$$\text{strong int.: } \rho \rightarrow 2\pi, \quad \tau \sim 10^{-22}\text{s}$$

$$\text{elmg. int.: } \pi \rightarrow 2\gamma, \quad \tau \sim 10^{-16}\text{s}$$

$$\text{weak int.: } \pi^- \rightarrow \mu^- + \bar{\nu}_\mu, \quad \tau \sim 10^{-8}\text{s}$$

$$\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu, \quad \tau \sim 10^{-6}\text{s}$$

- **lepton-number conservation:** $\mu^- \not\rightarrow e^- + \gamma$ (BR $\lesssim 10^{-11}$)

$\Rightarrow L_e, L_\mu, L_\tau$ individually conserved:

$$L_e = +1 \text{ for } e^-, \nu_e, \quad L_e = -1 \text{ for } e^+, \bar{\nu}_e, \quad \text{etc.}$$

(For massive ν 's with different masses, only $L_e + L_\mu + L_\tau$ is conserved.)

- **parity violation (Wu et al. 1957):**

e.g.: $K^+ \rightarrow \underbrace{2\pi, 3\pi}_{\text{final states of different parity}}$

$${}^{60}\text{Co} \rightarrow {}^{60}\text{Ni}^* + e^- + \bar{\nu}_e$$

\hookrightarrow polarization inversion does not yield inversion of spectra



The Fermi model

(Fermi 1933, further developed by Feynman, Gell-Mann and others after 1958)

Lagrangian for “current–current interaction” of four fermions:

$$\mathcal{L}_{\text{Fermi}}(x) = -2\sqrt{2}G_{\mu}J_{\rho}^{\dagger}(x)J^{\rho}(x), \quad G_{\mu} = 1.16639 \times 10^{-5} \text{ GeV}^{-2}$$

with $J_{\rho}(x) = J_{\rho}^{\text{lep}}(x) + J_{\rho}^{\text{had}}(x) = \text{charged weak current}$

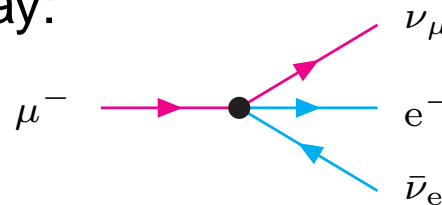
- Leptonic part J_{ρ}^{lep} of J_{ρ} :

$$J_{\rho}^{\text{lep}} = \overline{\psi_{\nu_e}}\gamma_{\rho}\omega_{-}\psi_e + \overline{\psi_{\nu_{\mu}}}\gamma_{\rho}\omega_{-}\psi_{\mu} \quad \omega_{\pm} = \frac{1}{2}(1 \pm \gamma_5) = \text{chirality projectors}$$

- ◇ only left-handed fermions ($\omega_{-}\psi$), right-handed anti-fermions ($\overline{\psi}\omega_{+}$) feel (charged-current) weak interactions \Rightarrow maximal P-violation

- ◇ doublet structure: $\begin{pmatrix} \nu_e \\ e^{-} \end{pmatrix}, \begin{pmatrix} \nu_{\mu} \\ \mu^{-} \end{pmatrix}$, later completed by $\begin{pmatrix} \nu_{\tau} \\ \tau^{-} \end{pmatrix}$

- ◇ $(J^{\text{lep},\rho})^{\dagger}J_{\rho}^{\text{lep}}$ induces muon decay:



- Hadronic part J_ρ^{had} of J_ρ :

Relevant quarks for energies $\lesssim 1 \text{ GeV}$: u, d, s, c

\hookrightarrow meson ($q\bar{q}$) and baryon (qqq) spectra

Question: doublet structure $\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}$?

Problem: e.g. annihilation of $u\bar{s}$ pair would not be allowed,

but is observed: $\underbrace{K^+}_{u\bar{s} \text{ pair in quark model}} \rightarrow \mu^+ \nu_\mu$

$u\bar{s}$ pair in quark model

Solution (Cabibbo 1963):

u-c-mixing and d-s-mixing in weak interaction

\hookrightarrow doublets $\begin{pmatrix} u \\ d' \end{pmatrix}, \begin{pmatrix} c \\ s' \end{pmatrix}$ with $\begin{pmatrix} d' \\ s' \end{pmatrix} = U_C \begin{pmatrix} d \\ s \end{pmatrix},$

orthogonal Cabibbo matrix $U_C = \begin{pmatrix} \cos \theta_C & \sin \theta_C \\ -\sin \theta_C & \cos \theta_C \end{pmatrix},$

empirical result: $\theta_C \approx 13^\circ$

$$J_\rho^{\text{had}} = \overline{\psi}_u \gamma_\rho \omega - \psi_{d'} + \overline{\psi}_c \gamma_\rho \omega - \psi_{s'}$$

Remarks on the Fermi model:

- **universal coupling** G_μ for all transitions
($U_C^\dagger U_C = 1$ is part of universality)
- no (pseudo-)scalar or tensor couplings, such as $(\bar{\psi}\psi)(\bar{\psi}\psi)$, $(\bar{\psi}\psi)(\bar{\psi}\gamma_5\psi)$, etc., necessary to describe low-energy experiments ($E \lesssim 1 \text{ GeV}$)
- **Problems:**
 - ◇ cross sections for $\nu_\mu e \rightarrow \nu_e \mu$, etc., grow for energy $E \rightarrow \infty$ as E^2
↪ **unitarity violation !**
 - ◇ no consistent evaluation of higher perturbative orders possible
(no cancellation of UV divergences)
↪ **non-renormalizability !**



“Intermediate-vector-boson (IVB) model”

Idea: “resolution” of four-fermion interaction by vector-boson exchange

Lagrangian:

$$\mathcal{L}_{\text{IVB}} = \mathcal{L}_{0,\text{ferm}} + \mathcal{L}_{0,W} + \mathcal{L}_{\text{int}},$$

$$\mathcal{L}_{0,\text{ferm}} = \overline{\psi}_f (i\cancel{\partial} - m_f) \psi_f, \quad (\text{summation over } f \text{ assumed})$$

$$\mathcal{L}_{0,W} = -\frac{1}{2} (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+) (\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) + M_W^2 W_\mu^+ W^{-,\mu},$$

$$\text{with } W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2), \quad W_\mu^i \text{ real}$$

W^\pm are vector bosons with electric charge $\pm e$ and mass M_W .

Propagator:
$$G_{\mu\nu}^{WW}(k) = \frac{-i}{k^2 - M_W^2} \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M_W^2} \right), \quad k = \text{momentum}$$

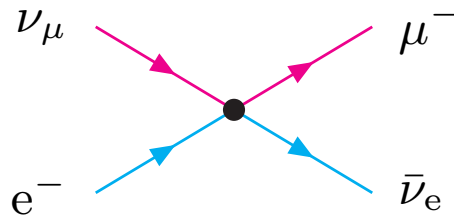
Interaction Lagrangian:
$$\mathcal{L}_{\text{int}} = \frac{g_W}{\sqrt{2}} (J^\rho W_\rho^+ + J^{\rho\dagger} W_\rho^-),$$

 $J^\rho = \text{charged weak current as in Fermi model}$



Four-fermion interaction in process $\nu_\mu e^- \rightarrow \mu^- \nu_e$

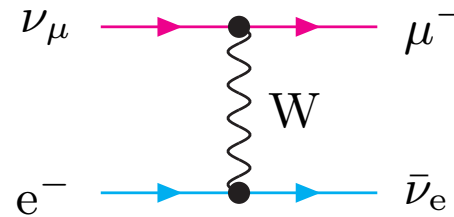
Fermi model:



$$-i2\sqrt{2}G_\mu g_{\rho\sigma}$$

$$\times [\bar{u}_\mu - \gamma^\rho \omega - u_{\nu_\mu}] [\bar{u}_{\nu_e} \gamma^\sigma \omega - u_{e^-}]$$

IVB model:



$$\frac{i}{2} g_W^2 \frac{1}{k^2 - M_W^2} \left(g_{\rho\sigma} - \frac{k_\rho k_\sigma}{M_W^2} \right)$$

$$\times [\bar{u}_\mu - \gamma^\rho \omega - u_{\nu_\mu}] [\bar{u}_{\nu_e} \gamma^\sigma \omega - u_{e^-}]$$

$$\Rightarrow \text{identification for } |k| \ll M_W: \quad 2\sqrt{2}G_\mu = \frac{g_W^2}{2M_W^2}$$

Consequences for the high-energy behaviour:

- k^ρ terms: $\bar{u}_{\nu_e} \not{k} \omega - u_{e^-} = \bar{u}_{\nu_e} (\not{p}_e - \not{p}_{\nu_e}) \omega - u_{e^-} = m_e \bar{u}_{\nu_e} \omega - u_{e^-}$
 \hookrightarrow no extra factors of scattering energy E
- propagator $1/(k^2 - M_W^2) \sim 1/E^2$ for $|k| \sim E \gg M_W$
 \hookrightarrow damping of amplitude in high-energy limit by factor $1/E^2$

$$\Rightarrow \text{cross section } \widetilde{E \rightarrow \infty} \text{ const}/E^2, \quad \Rightarrow \text{No unitarity violation !}$$

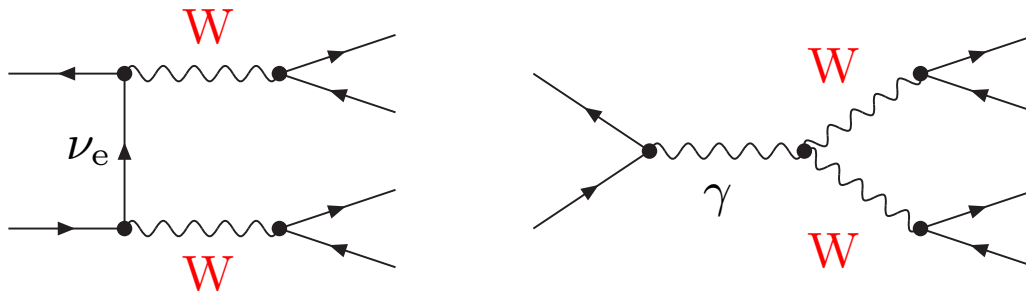
Comments on the IVB model:

- Formal similarity with QED interaction: $J^\rho W_\rho^+ + \text{h.c.} \longleftrightarrow j_{\text{elmg.}}^\rho A_\rho$
- Intermediate vector bosons can be produced, e.g.

$$\underbrace{u\bar{d}}_{\text{in pp collision}} \longrightarrow \underbrace{W^+ \rightarrow f\bar{f}'}_{W^\pm \text{ unstable}} \quad (\text{discovery 1983 at CERN})$$

Problems:

- ◇ **unitarity violations** in cross sections with longitudinal W bosons, e.g.



- ◇ **non-renormalizability**
(no consistent treatment of higher perturbative orders)

↔ **Solution by spontaneously broken gauge theories !**

2 The principle of local gauge invariance

QED as U(1) gauge theory:

Lagrangian $\mathcal{L}_{0,\text{ferm}} = \overline{\psi}_f (i\cancel{\partial} - m_f) \psi_f$ has **global phase symmetry**:

$$\psi_f \rightarrow \psi'_f = \exp\{-iQ_f e\theta\} \psi_f, \quad \overline{\psi}_f \rightarrow \overline{\psi}'_f = \overline{\psi}_f \exp\{+iQ_f e\theta\}$$

with **space-time-independent group parameter θ**

“Gauging the symmetry”: **demand local symmetry, $\theta \rightarrow \theta(x)$**

To maintain local symmetry, extend theory by **“minimal substitution”**:

$$\partial^\mu \rightarrow D^\mu = \partial^\mu + iQ_f e A^\mu(x) = \text{“covariant derivative”},$$
$$A^\mu(x) = \text{spin-1 gauge field (photon)}.$$

Transformation property of photon $A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu \theta(x)$ ensures

- $D_\mu \psi_f \rightarrow (D_\mu \psi_f)' = D'_\mu \psi'_f = \exp\{-iQ_f e\theta\} (D_\mu \psi_f)$
- gauge invariance of field-strength tensor $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Gauge-invariant Lagrangian of QED:

$$\mathcal{L}_{\text{QED}} = \overline{\psi}_f (i\cancel{\partial} - Q_f e \cancel{A} - m_f) \psi_f - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

Non-Abelian gauge theory (Yang–Mills theory):

Starting point:

Lagrangian $\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi)$ of free or self-interacting fields with “internal symmetry”:

- $\Phi = \begin{pmatrix} \phi_1 \\ \vdots \\ \phi_n \end{pmatrix}$ = multiplet of a compact Lie group G:

$$\Phi \rightarrow \Phi' = U(\theta)\Phi, \quad U(\theta) = \exp\{-igT^a\theta^a\} = \text{unitary},$$

$$T^a = \text{group generators}, \quad [T^a, T^b] = iC^{abc}T^c, \quad \text{Tr}\{T^a T^b\} = \frac{1}{2}\delta^{ab}$$

- \mathcal{L}_Φ is invariant under G: $\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi) = \mathcal{L}_\Phi(\Phi', \partial_\mu \Phi')$

Example: self-interacting (complex) boson multiplet

$$\mathcal{L}_\Phi = (\partial_\mu \Phi)^\dagger (\partial^\mu \Phi) - m^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2 \quad (m = \text{common boson mass}, \lambda = \text{coupling strength})$$

Gauging the symmetry by minimal substitution:

$$\mathcal{L}_\Phi(\Phi, \partial_\mu \Phi) \rightarrow \mathcal{L}_\Phi(\Phi, D_\mu \Phi) \quad \text{with } D_\mu = \partial_\mu + igT^a A_\mu^a(x),$$

g = gauge coupling, T^a = generator of G in Φ representation, $A_\mu^a(x)$ = gauge fields



Transformation property of gauge fields:

- $\mathcal{L}_\Phi(\Phi, D_\mu \Phi)$ local invariant if $D_\mu \Phi \rightarrow (D_\mu \Phi)' = D'_\mu \Phi' = U(\theta)(D_\mu \Phi)$
 $\Rightarrow T^a A'_\mu{}^a = UT^a A_\mu^a U^\dagger - \frac{i}{g} U(\partial_\mu U^\dagger), \quad A_\mu^a A^{a,\mu} = \text{not gauge invariant}$
infinitesimal form: $\delta A_\mu^a = gC^{abc} \delta\theta^b A_\mu^c + \partial_\mu \delta\theta^a$
- covariant definition of field strength: $[D_\mu, D_\nu] = igT^a F_{\mu\nu}^a$
 $\Rightarrow T^a F_{\mu\nu}^a \rightarrow T^a F'_{\mu\nu}{}^a = UT^a F_{\mu\nu}^a U^\dagger, \quad F_{\mu\nu}^a F^{a,\mu\nu} = \text{gauge invariant}$
explicit form: $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - gC^{abc} A_\mu^b A_\nu^c$

Yang–Mills Lagrangian for gauge and matter fields:

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4} F_{\mu\nu}^a F^{a,\mu\nu} + \mathcal{L}_\Phi(\Phi, D_\mu \Phi)$$

- Lagrangian contains terms of order $(\partial A)A^2, A^4$ in F^2 part
 \hookrightarrow cubic and quartic gauge-boson self-interactions
- gauge coupling determines gauge-boson–matter and gauge-boson self-interaction \rightarrow unification of interactions
- mass term $M^2(A_\mu^a A^{a,\mu})$ for gauge bosons forbidden by gauge invariance
 \hookrightarrow gauge bosons of unbroken Yang–Mills theory are massless

Quantum chromodynamics — gauge theory of strong interactions

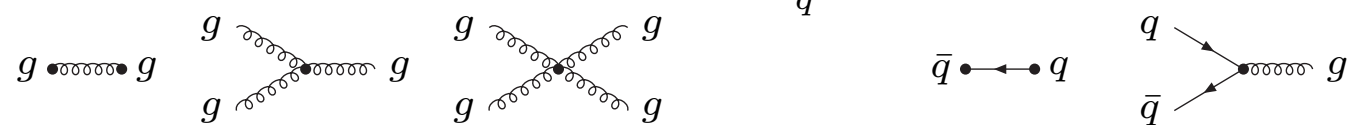
- **Gauge group:** $SU(3)_c$, $\dim. = 8$
structure constants f^{abc} , gauge coupling g_s , $\alpha_s = \frac{g_s^2}{4\pi}$
- **Gauge bosons:** 8 massless gluons g with fields $A_\mu^a(x)$, $a = 1, \dots, 8$
- **Matter fermions:** quarks q (spin- $\frac{1}{2}$) with flavours $q = d, u, s, c, b, t$ in fundamental representation:

$$\psi_q(x) \equiv q(x) = \begin{pmatrix} q_r(x) \\ q_g(x) \\ q_b(x) \end{pmatrix} = \text{colour triplet}$$

$$T^a = \frac{\lambda^a}{2}, \quad \text{Gell-Mann matrices } \lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \text{ etc.}$$

- **Lagrangian:**

$$\begin{aligned} \mathcal{L}_{\text{QCD}} &= -\frac{1}{4} F_{\mu\nu}^a F^{a,\mu\nu} + \sum_q \bar{\psi}_q (i\not{D} - m_q) \psi_q \\ &= -\frac{1}{4} \left(\partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_s f^{abc} A_\mu^b A_\nu^c \right)^2 + \sum_q \bar{\psi}_q \left(i\not{\partial} - g_s \frac{\lambda^a}{2} A^a - m_q \right) \psi_q \end{aligned}$$



3 The Standard Model of electroweak interaction (Glashow–Salam–Weinberg model)

3.1 The gauge group for electroweak interaction

Why unification of weak and elmg. interaction ?

- similarity: spin-1 fields couple to matter currents formed by spin- $\frac{1}{2}$ fields
- elmg. coupling of charged W^\pm bosons

γ, W^+, W^- as gauge bosons of group $SU(2)$? – No!

Reason: charge operator Q cannot be $SU(2)$ generator, since $\text{Tr} \{Q\} \neq 0$

for fermion doublets: $Q = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}$ for $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$, etc.

Possible way out: additional heavy fermions like E^+ as partner to e^- ?

↪ no experimental confirmation !

Minimal solution: $SU(2)_I \times U(1)_Y$

- $SU(2)_I$ → weak isospin group with gauge bosons W^+ , W^- , W^0
- $U(1)_Y$ → weak hypercharge with gauge boson B

W^0 and B carry identical quantum numbers

↔ two neutral gauge bosons γ , Z as mixed states

Experiment: 1973 discovery of neutral weak currents at CERN

↔ indirect confirmation of Z exchange

1983 discovery of W^\pm and Z bosons at CERN

3.2 Fermion sector and minimal substitution

Multiplet structure:

Distinguish between left-/right-handed parts of fermions: $\psi^L = \omega_- \psi$, $\psi^R = \omega_+ \psi$

- ψ^L couple to W^\pm → group ψ^L into $SU(2)_I$ doublets, weak isospin $T_I^a = \frac{\sigma^a}{2}$
- ψ^R do not couple to W^\pm → ψ^R are $SU(2)_I$ singlets, weak isospin $T_I^a = 0$
- $\psi^{L/R}$ couple to γ in the same way
 ↪ adjust coupling to $U(1)_Y$ (i.e. fix weak hypercharges $Y^{L/R}$ for $\psi^{L/R}$)
 such that elmg. coupling results: $\mathcal{L}_{\text{int,QED}} = -Q_f e \bar{\psi}_f \not{A} \psi_f$

Fermion content of the SM:

(ignoring possible right-handed neutrinos)

				T_I^3	Q	
leptons:	$\Psi_L^L =$	$\begin{pmatrix} \nu_e^L \\ e^L \end{pmatrix},$	$\begin{pmatrix} \nu_\mu^L \\ \mu^L \end{pmatrix},$	$\begin{pmatrix} \nu_\tau^L \\ \tau^L \end{pmatrix},$	$+\frac{1}{2}$	0
					$-\frac{1}{2}$	-1
	$\psi_i^R =$	$e^R,$	$\mu^R,$	$\tau^R,$	0	-1
quarks:	$\Psi_Q^L =$	$\begin{pmatrix} u^L \\ d^L \end{pmatrix},$	$\begin{pmatrix} c^L \\ s^L \end{pmatrix},$	$\begin{pmatrix} t^L \\ b^L \end{pmatrix},$	$+\frac{1}{2}$	$+\frac{2}{3}$
					$-\frac{1}{2}$	$-\frac{1}{3}$
(Each quark exists in 3 colours!)	$\psi_u^R =$	$u^R,$	$c^R,$	$t^R,$	0	$+\frac{2}{3}$
	$\psi_d^R =$	$d^R,$	$s^R,$	$b^R,$	0	$-\frac{1}{3}$

Free Lagrangian of (still massless) fermions:

$$\mathcal{L}_{0,\text{ferm}} = i\overline{\psi}_f \not{\partial} \psi_f = i\overline{\Psi}_L^L \not{\partial} \Psi_L^L + i\overline{\Psi}_Q^L \not{\partial} \Psi_Q^L + i\overline{\psi}_l^R \not{\partial} \psi_l^R + i\overline{\psi}_u^R \not{\partial} \psi_u^R + i\overline{\psi}_d^R \not{\partial} \psi_d^R$$

Minimal substitution:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ig_2 T_1^a W_\mu^a + ig_1 \frac{1}{2} Y B_\mu = D_\mu^L \omega_- + D_\mu^R \omega_+,$$

$$D_\mu^L = \partial_\mu - \frac{ig_2}{\sqrt{2}} \begin{pmatrix} 0 & W_\mu^+ \\ W_\mu^- & 0 \end{pmatrix} - \frac{i}{2} \begin{pmatrix} g_2 W_\mu^3 - g_1 Y^L B_\mu & 0 \\ 0 & -g_2 W_\mu^3 - g_1 Y^L B_\mu \end{pmatrix},$$

$$D_\mu^R = \partial_\mu + ig_1 \frac{1}{2} Y^R B_\mu$$

Photon identification:

“Weinberg rotation”: $\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} = \begin{pmatrix} c_W & s_W \\ -s_W & c_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix},$ $c_W = \cos \theta_W, s_W = \sin \theta_W,$
 $\theta_W = \text{weak mixing angle}$

$$D_\mu^L \Big|_{A_\mu} = -\frac{i}{2} A_\mu \begin{pmatrix} -g_2 s_W - g_1 c_W Y^L & 0 \\ 0 & g_2 s_W - g_1 c_W Y^L \end{pmatrix} \stackrel{!}{=} ie A_\mu \begin{pmatrix} Q_1 & 0 \\ 0 & Q_2 \end{pmatrix}$$

- charged difference in doublet $Q_1 - Q_2 = 1 \rightarrow g_2 = \frac{e}{s_W}$
- normalize $Y^{L/R}$ such that $g_1 = \frac{e}{c_W}$
- $\hookrightarrow Y$ fixed by “Gell-Mann–Nishijima relation”: $Q = T_1^3 + \frac{Y}{2}$

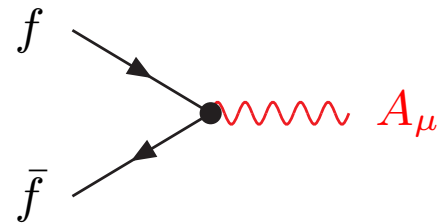
Fermion–gauge-boson interaction:

$$\mathcal{L}_{\text{ferm, YM}} = \frac{e}{\sqrt{2}s_W} \bar{\Psi}_F^L \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix} \Psi_F^L + \frac{e}{2c_W s_W} \bar{\Psi}_F^L \sigma^3 Z \Psi_F^L \\ - e \frac{s_W}{c_W} Q_f \bar{\psi}_f Z \psi_f - e Q_f \bar{\psi}_f A \psi_f \quad (f=\text{all fermions}, F=\text{all doublets})$$

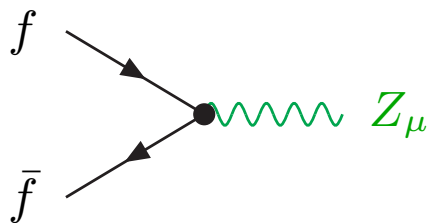
Feynman rules:



$$\frac{ie}{\sqrt{2}s_W} \gamma_\mu \omega_-$$



$$-iQ_f e \gamma_\mu$$



$$ie\gamma_\mu (g_f^+ \omega_+ + g_f^- \omega_-) = ie\gamma_\mu (v_f - a_f \gamma_5)$$

$$\text{with } g_f^+ = -\frac{s_W}{c_W} Q_f, \quad g_f^- = -\frac{s_W}{c_W} Q_f + \frac{T_{I,f}^3}{c_W s_W},$$

$$v_f = -\frac{s_W}{c_W} Q_f + \frac{T_{I,f}^3}{2c_W s_W}, \quad a_f = \frac{T_{I,f}^3}{2c_W s_W}$$

3.3 Gauge-boson sector

Yang–Mills Lagrangian for gauge fields:

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4}W_{\mu\nu}^a W^{a,\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

Field-strength tensors:

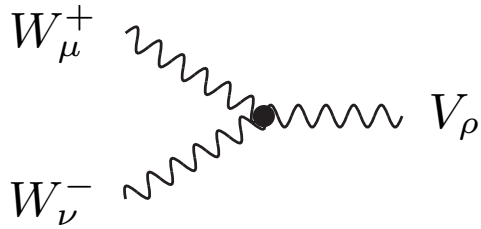
$$W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g_2 \epsilon^{abc} W_\mu^b W_\nu^c, \quad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

Lagrangian in terms of “physical” fields:

$$\begin{aligned} \mathcal{L}_{\text{YM}} = & -\frac{1}{2}(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)(\partial^\mu W^{-,\nu} - \partial^\nu W^{-,\mu}) \\ & - \frac{1}{4}(\partial_\mu Z_\nu - \partial_\nu Z_\mu)(\partial^\mu Z^\nu - \partial^\nu Z^\mu) - \frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \\ & + \text{(trilinear interaction terms involving } AW^+W^-, ZW^+W^-) \\ & + \text{(quadrilinear interaction terms involving} \\ & \quad AAW^+W^-, AZW^+W^-, ZZW^+W^-, W^+W^-W^+W^-) \end{aligned}$$

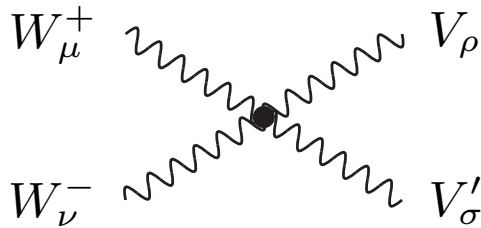
Feynman rules for gauge-boson self-interactions:

(fields and momenta incoming)



$$ieC_{WWV} \left[g_{\mu\nu}(k_+ - k_-)_\rho + g_{\nu\rho}(k_- - k_V)_\mu + g_{\rho\mu}(k_V - k_+)_\nu \right]$$

$$\text{with } C_{WW\gamma} = 1, \quad C_{WWZ} = -\frac{c_W}{s_W}$$



$$ie^2 C_{WWVV'} \left[2g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\sigma\nu} - g_{\mu\sigma}g_{\nu\rho} \right]$$

$$\text{with } C_{WW\gamma\gamma} = -1, \quad C_{WW\gamma Z} = \frac{c_W}{s_W},$$

$$C_{WWZZ} = -\frac{c_W^2}{s_W^2}, \quad C_{WWWW} = \frac{1}{s_W^2}$$

3.4 Higgs sector and spontaneous symmetry breaking

Idea: spontaneous breakdown of $SU(2)_I \times U(1)_Y$ symmetry $\rightarrow U(1)_{\text{em}} \text{g}$ symmetry

\hookrightarrow masses for W^\pm and Z bosons, but γ remains massless

Note: choice of scalar extension of massless model involves freedom

GSW model:

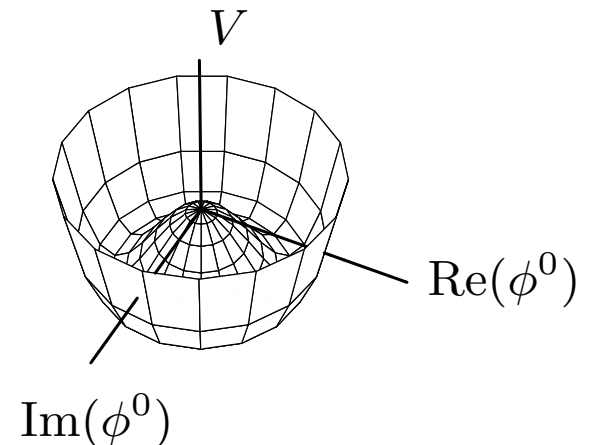
Minimal scalar sector with complex scalar doublet $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$, $Y_\Phi = 1$

Scalar self-interaction via Higgs potential:

$$V(\Phi) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{4} (\Phi^\dagger \Phi)^2, \quad \mu^2, \lambda > 0,$$

$= SU(2)_I \times U(1)_Y$ symmetric

$$V(\Phi) = \text{minimal for } |\Phi| = \sqrt{\frac{2\mu^2}{\lambda}} \equiv \frac{v}{\sqrt{2}} > 0$$



ground state Φ_0 (=vacuum expectation value of Φ) not unique

specific choice $\Phi_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$ not gauge invariant \Rightarrow spontaneous symmetry breaking

emg. gauge invariance unbroken, since $Q\Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Phi_0 = 0$

Field excitations in Φ :

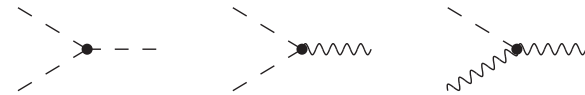
$$\Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}} \left(v + H(x) + i\chi(x) \right) \end{pmatrix}$$

Gauge-invariant Lagrangian of Higgs sector: $(\phi^- = (\phi^+)^\dagger)$

$$\mathcal{L}_H = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\Phi) \quad \text{with } D_\mu = \partial_\mu - ig_2 \frac{\sigma^a}{2} W_\mu^a + i \frac{g_1}{2} B_\mu$$

$$\begin{aligned} &= (\partial_\mu \phi^+) (\partial^\mu \phi^-) - \frac{iev}{2s_W} (W_\mu^+ \partial^\mu \phi^- - W_\mu^- \partial^\mu \phi^+) + \frac{e^2 v^2}{4s_W^2} W_\mu^+ W^{-,\mu} \\ &+ \frac{1}{2} (\partial\chi)^2 + \frac{ev}{2c_W s_W} Z_\mu \partial^\mu \chi + \frac{e^2 v^2}{4c_W^2 s_W^2} Z^2 + \frac{1}{2} (\partial H)^2 - \mu^2 H^2 \end{aligned}$$

+ (trilinear SSS , SSV , SVV interactions)



+ (quadrilinear $SSSS$, $SSVV$ interactions)



Implications:

- gauge-boson masses: $M_W = \frac{ev}{2s_W}$, $M_Z = \frac{ev}{2c_W s_W} = \frac{M_W}{c_W}$, $M_\gamma = 0$
- physical Higgs boson H: $M_H = \sqrt{2\mu^2}$ = free parameter
- would-be Goldstone bosons ϕ^\pm, χ : unphysical degrees of freedom

3.5 Fermion masses and Yukawa couplings

Ordinary Dirac mass terms $m_f \overline{\psi}_f \psi_f = m_f (\overline{\psi}_f^L \psi_f^R + \overline{\psi}_f^R \psi_f^L)$ not gauge invariant

↪ introduce fermion masses by (gauge-invariant) Yukawa interaction

Lagrangian for Yukawa couplings:

$$\mathcal{L}_{\text{Yuk}} = -\overline{\Psi}_L^L G_l \psi_l^R \Phi - \overline{\Psi}_Q^L G_u \psi_u^R \tilde{\Phi} - \overline{\Psi}_Q^L G_d \psi_d^R \Phi + \text{h.c.}$$

- $G_l, G_u, G_d = 3 \times 3$ matrices in 3-dim. space of generations (ν masses ignored)
- $\tilde{\Phi} = i\sigma^2 \Phi^* = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$ = charge conjugate Higgs doublet, $Y_{\tilde{\Phi}} = -1$

Fermion mass terms:

mass terms = bilinear terms in \mathcal{L}_{Yuk} , obtained by setting $\Phi \rightarrow \Phi_0$:

$$\mathcal{L}_{m_f} = -\frac{v}{\sqrt{2}} \overline{\psi}_l^L G_l \psi_l^R - \frac{v}{\sqrt{2}} \overline{\psi}_u^L G_u \psi_u^R - \frac{v}{\sqrt{2}} \overline{\psi}_d^L G_d \psi_d^R + \text{h.c.}$$

↪ diagonalization by unitary field transformations ($f = l, u, d$)

$$\hat{\psi}_f^{L/R} \equiv U_f^{L/R} \psi_f^{L/R} \quad \text{such that} \quad \frac{v}{\sqrt{2}} U_f^L G_f (U_f^R)^\dagger = \text{diag}(m_f)$$

$$\Rightarrow \text{standard form:} \quad \mathcal{L}_{m_f} = -m_f \overline{\hat{\psi}}_f^L \hat{\psi}_f^R + \text{h.c.} = -m_f \overline{\hat{\psi}}_f \hat{\psi}_f$$

Quark mixing:

- ψ_f correspond to eigenstates of the gauge interaction
- $\hat{\psi}_f$ correspond to mass eigenstates,
for **massless neutrinos** define $\hat{\psi}_\nu^L \equiv U_l^L \psi_\nu^L \rightarrow$ **no lepton-flavour changing**

Yukawa and gauge interactions in terms of mass eigenstates:

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} = & -\frac{\sqrt{2}m_l}{v} \left(\phi^+ \overline{\hat{\psi}_{\nu_l}^L} \hat{\psi}_l^R + \phi^- \overline{\hat{\psi}_l^R} \hat{\psi}_{\nu_l}^L \right) + \frac{\sqrt{2}m_u}{v} \left(\phi^+ \overline{\hat{\psi}_u^R} V \hat{\psi}_d^L + \phi^- \overline{\hat{\psi}_d^L} V^\dagger \hat{\psi}_u^R \right) \\ & - \frac{\sqrt{2}m_d}{v} \left(\phi^+ \overline{\hat{\psi}_u^L} V \hat{\psi}_d^R + \phi^- \overline{\hat{\psi}_d^R} V^\dagger \hat{\psi}_u^L \right) - \frac{m_f}{v} i \text{sgn}(T_{I,f}^3) \chi \overline{\hat{\psi}_f} \gamma_5 \hat{\psi}_f \\ & - \frac{m_f}{v} (v + H) \overline{\hat{\psi}_f} \hat{\psi}_f, \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{ferm, YM}} = & \frac{e}{\sqrt{2}s_W} \overline{\hat{\Psi}_L^L} \begin{pmatrix} 0 & W^+ \\ W^- & 0 \end{pmatrix} \hat{\psi}_L^L + \frac{e}{\sqrt{2}s_W} \overline{\hat{\Psi}_Q^L} \begin{pmatrix} 0 & V W^+ \\ V^\dagger W^- & 0 \end{pmatrix} \hat{\psi}_Q^L \\ & + \frac{e}{2c_W s_W} \overline{\hat{\Psi}_F^L} \sigma^3 \not{Z} \hat{\Psi}_F^L - e \frac{s_W}{c_W} Q_f \overline{\hat{\psi}_f} \not{Z} \hat{\psi}_f - e Q_f \overline{\hat{\psi}_f} \not{A} \hat{\psi}_f \end{aligned}$$

- only charged-current coupling of quarks modified by $V = U_u^L (U_d^L)^\dagger =$ **unitary**
(Cabibbo–Kobayashi–Maskawa (CKM) matrix)
- **Higgs–fermion coupling strength** = $\frac{m_f}{v}$

Features of the CKM mixing:

- $V = 3$ -dim. generalization of Cabibbo matrix U_C
- V is parametrized by 4 free parameters: 3 real angles, 1 complex phase
 \leftrightarrow complex phase is the only source of CP violation in SM

counting:

$$\begin{aligned} & \left(\begin{array}{l} \text{\#real d.o.f.} \\ \text{in } V \end{array} \right) - \left(\begin{array}{l} \text{\#unitarity} \\ \text{relations} \end{array} \right) - \left(\begin{array}{l} \text{\#phase diffs. of} \\ u\text{-type quarks} \end{array} \right) - \left(\begin{array}{l} \text{\#phase diffs. of} \\ d\text{-type quarks} \end{array} \right) - \left(\begin{array}{l} \text{\#phase diff. between} \\ u\text{- and } d\text{-type quarks} \end{array} \right) \\ & = 18 - 9 - 2 - 2 - 1 = 4 \end{aligned}$$

- no flavour-changing neutral currents in lowest order,
flavour-changing suppressed by factors $G_\mu(m_{q_1}^2 - m_{q_2}^2)$ in higher orders
("Glashow–Iliopoulos–Maiani mechanism")

3.6 Quantization — gauge fixing and Faddeev–Popov sector

Gauge fields contain unphysical degrees of freedom that must not be quantized.

Consequences:

- gauge-boson propagators ill-defined without gauge fixing, e.g. for photon the (singular) operator $(g_{\mu\nu}\square - \partial_\mu\partial_\nu)$ would have to be inverted
- in path integral $\int \mathcal{D}A_\mu^a \exp\{i \int dx \mathcal{L}\}$: only one representative of each gauge orbit should contribute, otherwise integral over gauge-equivalent fields diverges
 - ↪ fix gauge by δ -functions $\delta(F^a[A_\mu^a] - C^a)$ in path integral ($C^a = \text{const.}$)
 - ↪ by averaging over C^a , gauge fixing can be cast in terms of a **gauge-fixing Lagrangian \mathcal{L}_{gf}**

Gauge-fixing Lagrangian of general R_ξ gauge:

$$\mathcal{L}_{\text{gf}} = -\frac{1}{\xi_W} F^\pm F^\mp - \frac{1}{2\xi_Z} (F^Z)^2 - \frac{1}{2\xi_\gamma} (F^\gamma)^2$$

with the **gauge-fixing functionals F^a** : ($\xi_V =$ arbitrary gauge-fixing parameters)

$$F^\pm = \partial W^\pm \mp i\xi_W M_W \phi^\pm, \quad F^Z = \partial Z - \xi_Z M_Z \chi, \quad F^\gamma = \partial A$$

Features of the R_ξ gauge fixing:

- elimination of mixing terms $(W_\mu^\pm \partial^\mu \phi^\mp)$, $(Z_\mu \partial^\mu \chi)$ in Lagrangian
 \hookrightarrow decoupling of gauge and would-be Goldstone fields (no mix propagators)
- boson propagators:

$$\begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} \begin{array}{c} V \\ \text{---} \\ \bullet \end{array} \begin{array}{c} \rightarrow \\ k \end{array} \quad G_{\mu\nu}^{VV}(k) = -i \left[\frac{g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2 - M_V^2} + \frac{k_\mu k_\nu}{k^2} \frac{\xi_V}{k^2 - \xi_V M_V^2} \right], \quad V = W, Z, \gamma$$

$$\begin{array}{c} \bullet \\ \text{---} \\ \bullet \end{array} \begin{array}{c} S \\ \text{---} \\ \bullet \end{array} \begin{array}{c} \rightarrow \\ k \end{array} \quad G^{SS}(k) = \frac{i}{k^2 - \xi_V M_V^2}, \quad S = \phi, \chi$$

- important special cases:

◇ $\xi_V = 1$: ‘t Hooft–Feynman gauge

\hookrightarrow convenient gauge-boson propagators $\frac{-ig_{\mu\nu}}{k^2 - M_V^2}$

◇ $\xi_W, \xi_Z \rightarrow \infty$: “unitary gauge”

\hookrightarrow elimination of would-be Goldstone bosons

Faddeev–Popov ghosts

Consistent use of gauge fixing in path integral: **Faddeev–Popov ansatz**

$$\int \mathcal{D}A_\mu^a \exp \left\{ i \int dx \mathcal{L} \right\} \delta(F^a[A_\mu^a] - C^a) \det \left(\frac{\delta F^a}{\delta \theta^b} \right)$$

with the gauge variation of the functionals F^a :

$$\left(\frac{\delta F^a(x)}{\delta \theta^b(y)} \right) = M^{ab}(x) \delta(x - y),$$

$$M^{VV'}(x) = \delta^{VV'} (\square_x + \xi_V M_V^2) + \text{terms linear in vector and scalar fields}$$

Functional determinant can be written as path integral over

Grassmann-valued auxiliary fields $u^a(x), \bar{u}^a(x)$: (Faddeev–Popov ghost fields)

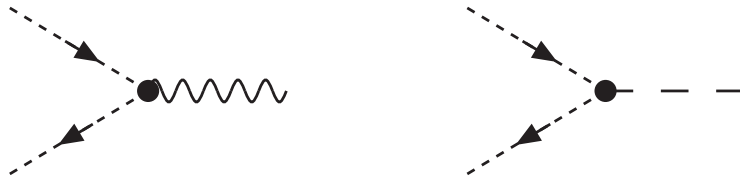
$$\det \left(\frac{\delta F^a}{\delta \theta^b} \right) \propto \int \mathcal{D}u^a \int \mathcal{D}\bar{u}^a \exp \left\{ i \int dx \mathcal{L}_{\text{FP}} \right\}$$

$$\mathcal{L}_{\text{FP}}(x) = -\bar{u}^a(x) M^{ab}(x) u^b(x) = -\bar{u}^V (\square + \xi_V M_V^2) u^V + \dots$$

↪ ghost propagators: $u^V \bullet \xrightarrow[k]{\dots} \bullet \bar{u}^V$ $\frac{i}{k^2 - \xi_V M_V^2}$

Features of the Faddeev–Popov ghost fields:

- **ghosts do not correspond to physical states**
(ghost propagators have poles at unphysical mass values $\xi_V M_V^2$)
↪ appear only inside loops in diagrams for physical processes
- ghost fields have spin 0, but are anti-commuting
(would violate spin-statistics theorem as physical states)
↪ **signs as for fermions in Feynman rules**
- ghost fields couple to gauge and scalar fields (not to fermions):



4 Electroweak phenomenology

4.1 Brief overview

Features of the electroweak Standard Model

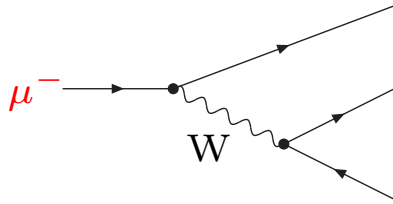
- **Higgs boson not yet found**, particle content verified otherwise
 - **No really significant contradictions** of GSW model with experiment
 - Input parameters:
$$\alpha = \frac{e^2}{4\pi} \approx 1/137, \quad M_W \approx 80 \text{ GeV}, \quad M_Z \approx 91 \text{ GeV}, \quad M_H \gtrsim 100 \text{ GeV}, \quad m_f, \quad V$$
 - **GSW model = consistent quantum field theory**
 - ◇ matrix elements respect unitarity
 - ◇ renormalizability
- ⇒ evaluation of higher perturbative orders possible
(and phenomenologically necessary !)

Important electroweak experiments

- **Muon decay:**

$$\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e$$

determination of the **Fermi constant**

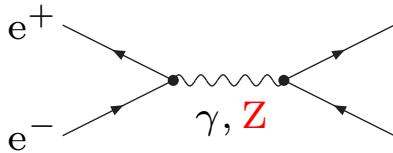


$$G_\mu = \frac{\pi\alpha M_Z^2}{\sqrt{2}M_W^2(M_Z^2 - M_W^2)} + \dots$$

- **Z production (LEP1/SLC):**

$$e^+e^- \rightarrow Z \rightarrow f\bar{f}$$

various precision measurements at the Z resonance: $M_Z, \Gamma_Z, \sigma_{\text{had}}, A_{\text{FB}}, A_{\text{LR}}, \text{etc.}$



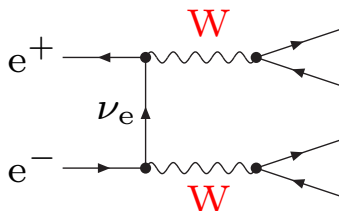
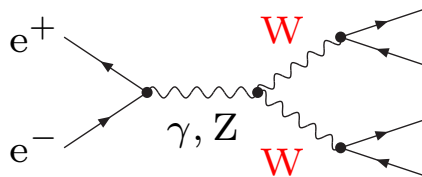
⇒ **good knowledge of the $Zf\bar{f}$ sector**

- **W-pair production (LEP2/ILC):** $e^+e^- \rightarrow WW \rightarrow 4f(+\gamma)$

– measurement of M_W

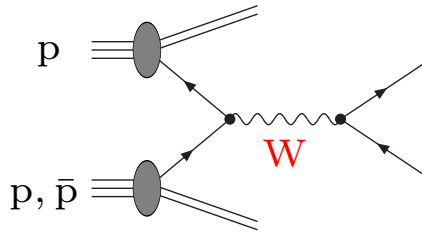
– $\gamma WW/ZWW$ couplings

– quartic couplings: $\gamma\gamma WW, \gamma ZWW$



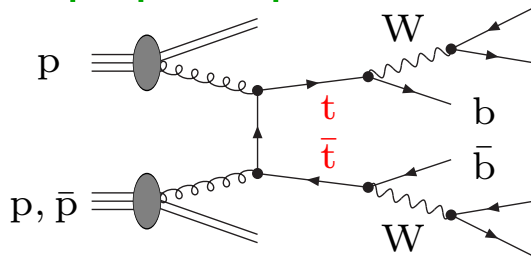
Important electroweak experiments (continued)

- **W production** (Tevatron/LHC): $pp, p\bar{p} \rightarrow W \rightarrow l\nu_l(+\gamma)$



- measurement of M_W
- bounds on γWW coupling

- **top-quark production** (Tevatron/LHC): $pp, p\bar{p} \rightarrow t\bar{t} \rightarrow 6f$



- measurement of m_t

Theoretical predictions

parametrized by $\alpha(M_Z)$, M_W , M_Z , m_t , m_f , $\alpha_s(M_Z)$ and M_H

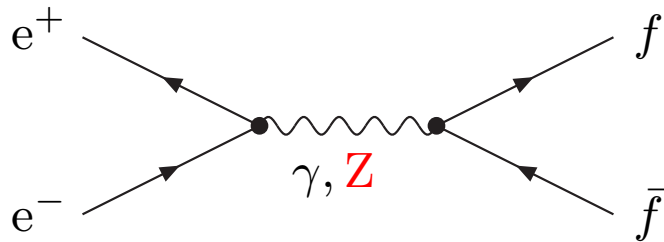
↪ global fit of SM to data yields bounds on M_H

But: high precision necessary,
since M_H sensitivity weak

$$\sim \frac{\alpha}{\pi} \log(M_H/M_W)$$

4.2 Z-boson physics at LEP1 and SLC

Precision study of the Z line shape

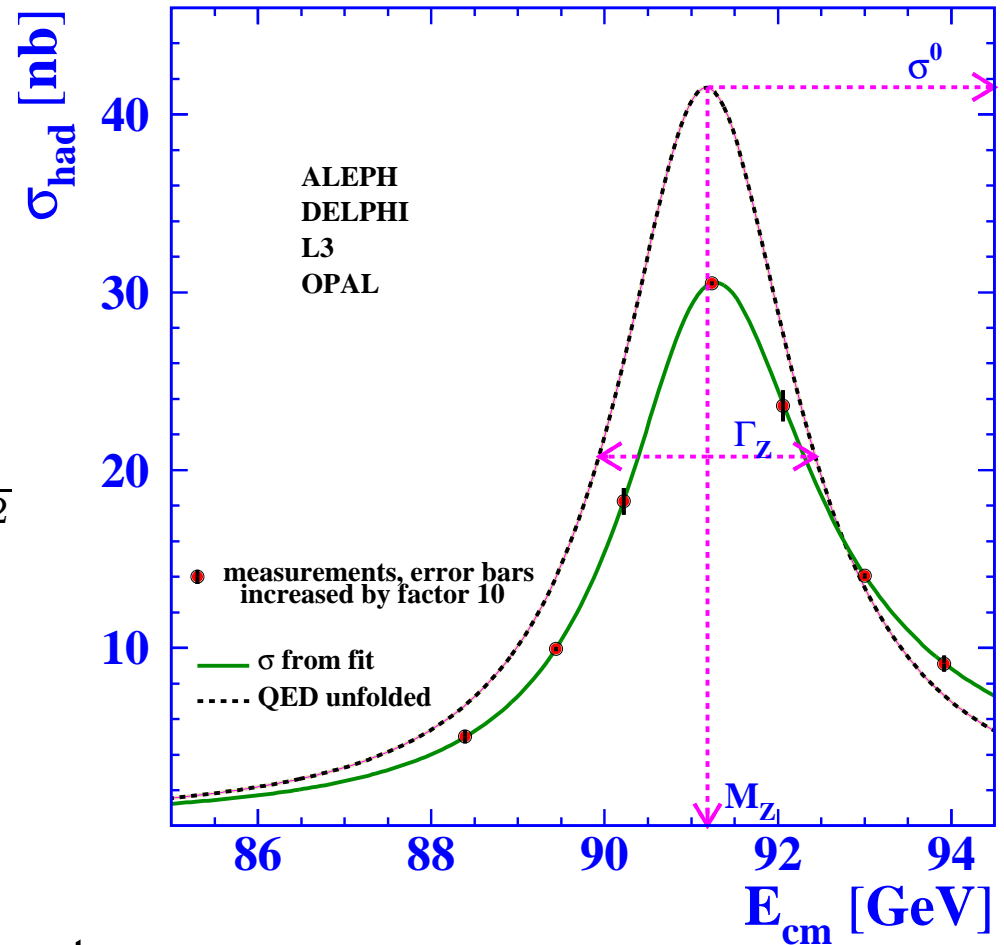


Unfolded resonance:

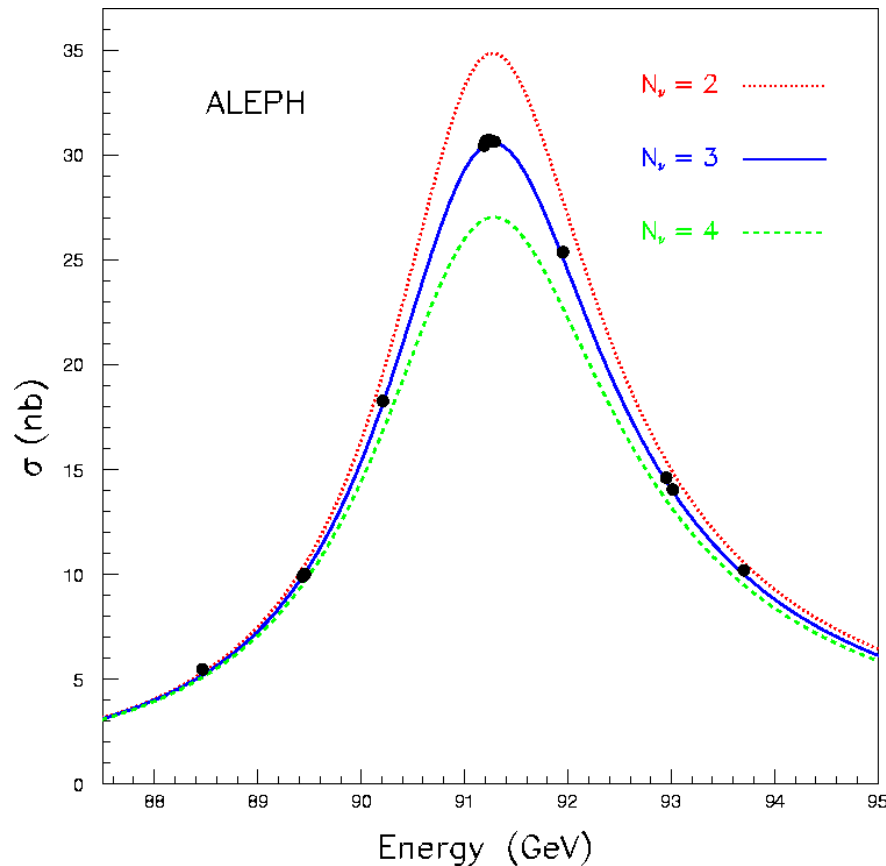
$$\sigma_{\text{res}}(s) = \sigma^0 \frac{s \Gamma_Z^2}{\left| s - M_Z^2 + i M_Z \Gamma_Z \frac{s}{M_Z^2} \right|^2}$$

Resonance observables:

- **Z mass** and **width**: M_Z, Γ_Z
- **peak cross section**: σ_{had}^0
- various asymmetries: $A_{\text{FB}}, A_{\text{LR}}$, etc.
- ratios of decay widths: $R_l = \frac{\Gamma_{\text{had}}}{\Gamma_l}$, etc.



Number of light neutrinos



$$\Gamma_Z = \Gamma_{\text{had}} + \Gamma_e + \Gamma_\mu + \Gamma_\tau + \Gamma_{\text{inv}}$$

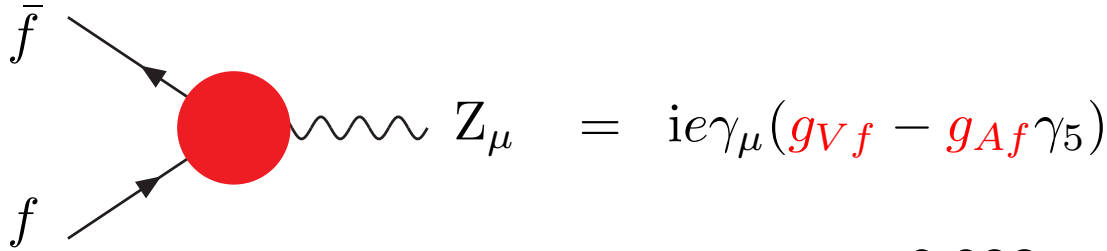
- Γ_Z measured from Z line shape
- Γ_{had} and $\Gamma_{l=e,\mu,\tau}$ from

$$R_l = \frac{\Gamma_{\text{had}}}{\Gamma_l} \quad \text{and} \quad \sigma_{\text{had}}^0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_{\text{had}}}{\Gamma_Z^2}$$

Fit of Γ_Z , R_l , and σ_{had}^0 yields invisible Z-decay width: $\Gamma_{\text{inv}} = N_\nu \Gamma_{Z \rightarrow \nu \bar{\nu}}^{\text{theory}}$

$$\hookrightarrow N_\nu = 2.9840 \pm 0.0082$$

Effective Z-boson–fermion couplings



Leptonic couplings from LEP1
asymmetry measurements, e.g.:

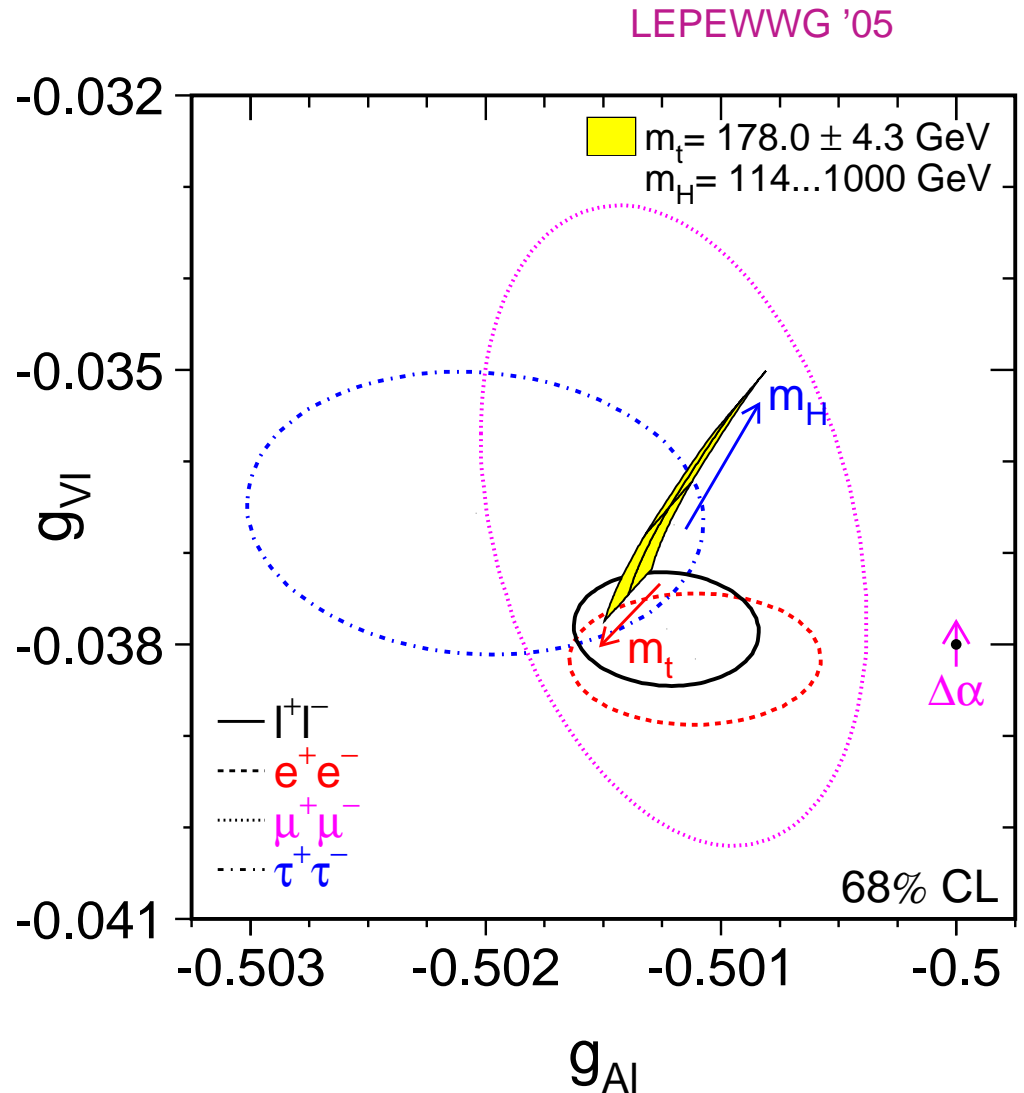
$$A_{\text{FB}}^{0,f} = \frac{\sigma_{f,\text{F}}^0 - \sigma_{f,\text{B}}^0}{\sigma_{f,\text{F}}^0 + \sigma_{f,\text{B}}^0} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f$$

(F/B = For/Backward hemisphere)

$$\text{with } \mathcal{A}_f = \frac{2g_V f g_A f}{g_V^2 f + g_A^2 f}$$

Good agreement with SM

- lepton universality confirmed
- constraints on m_t and M_H



Translation of effective couplings into effective weak mixing angle

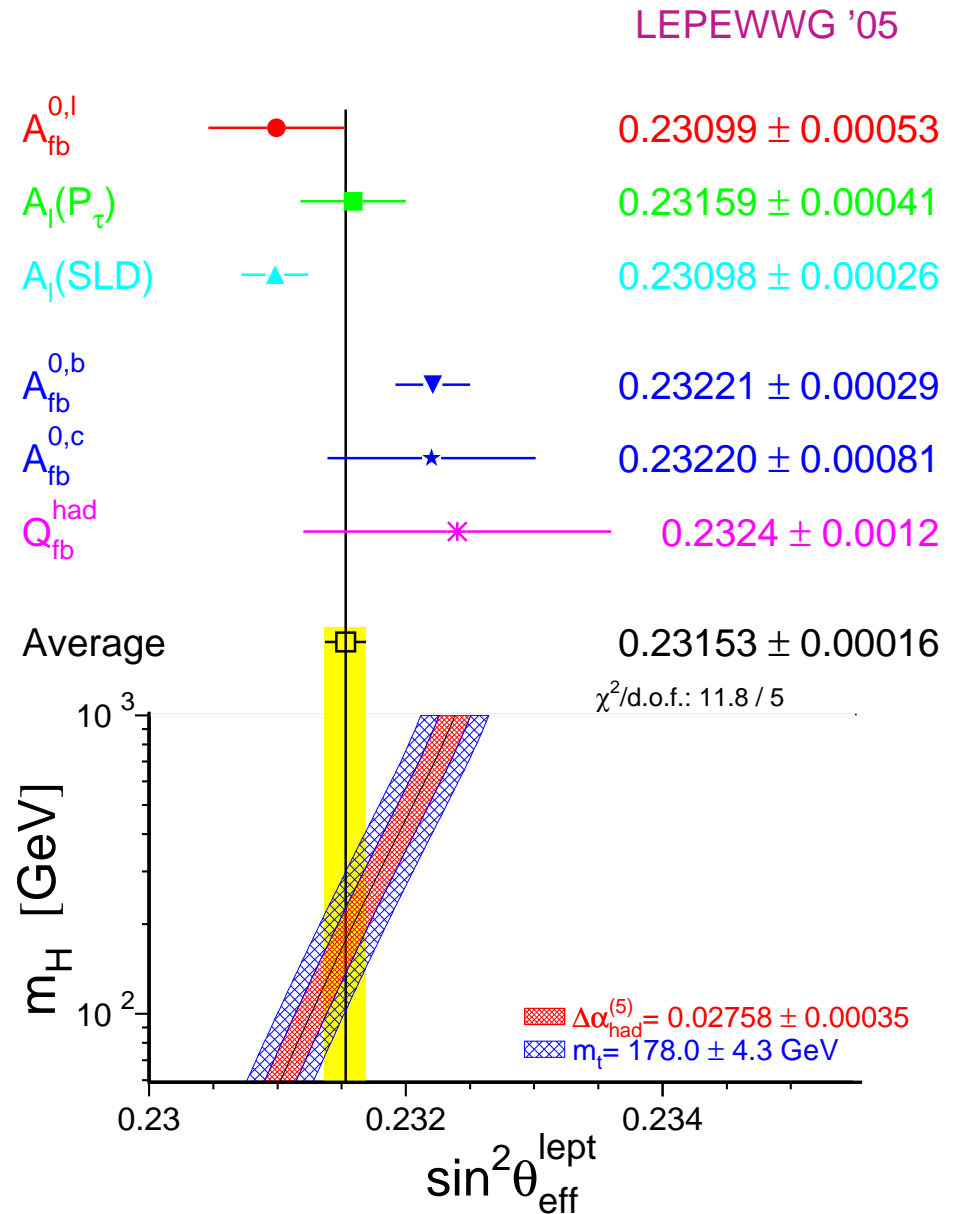
$$\sin^2 \theta_{\text{eff}}^{\text{lept}} = \frac{1}{4} \left(1 - \text{Re} \left\{ \frac{g_{Vl}}{g_{Al}} \right\} \right)$$

Important features:

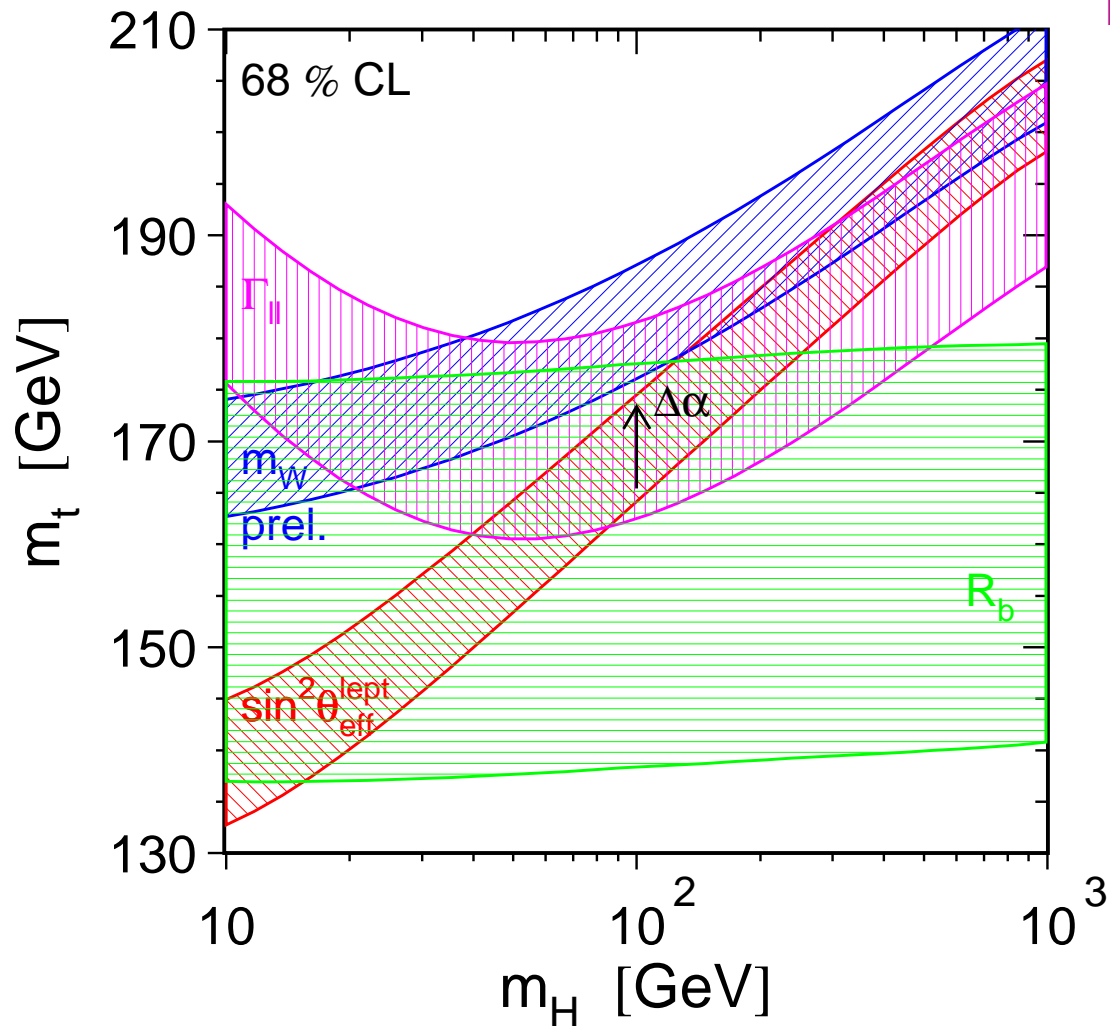
- high sensitivity to M_H
- combination of very different observables
- $\sim 3\sigma$ difference between $A_{\text{FB}}^{0,b}$ (LEP) and $A_{\text{LR}}^{0,l}$ (SLD)

with the initial-state pol. asymmetry

$$A_{\text{LR}}^{0,l} = \frac{\sigma_{\text{L}}^0 - \sigma_{\text{R}}^0}{\sigma_{\text{L}}^0 + \sigma_{\text{R}}^0} \frac{1}{\langle |\mathcal{P}_e| \rangle}$$

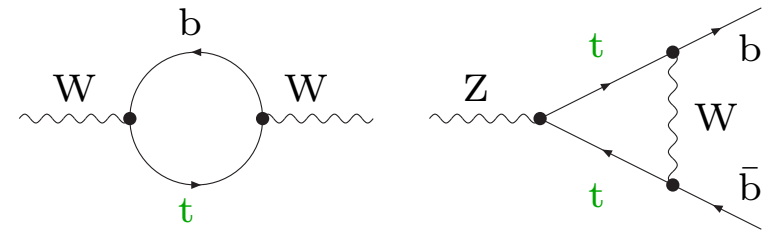


Observables most sensitive to m_t and M_H

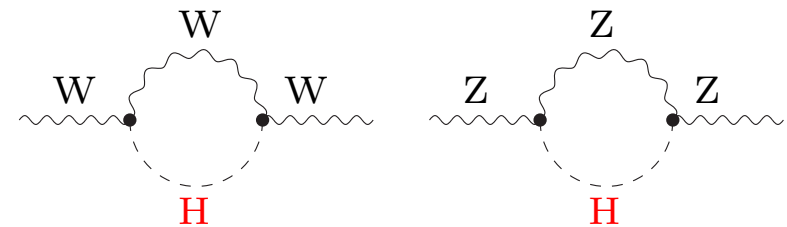


LEPEWWG '05

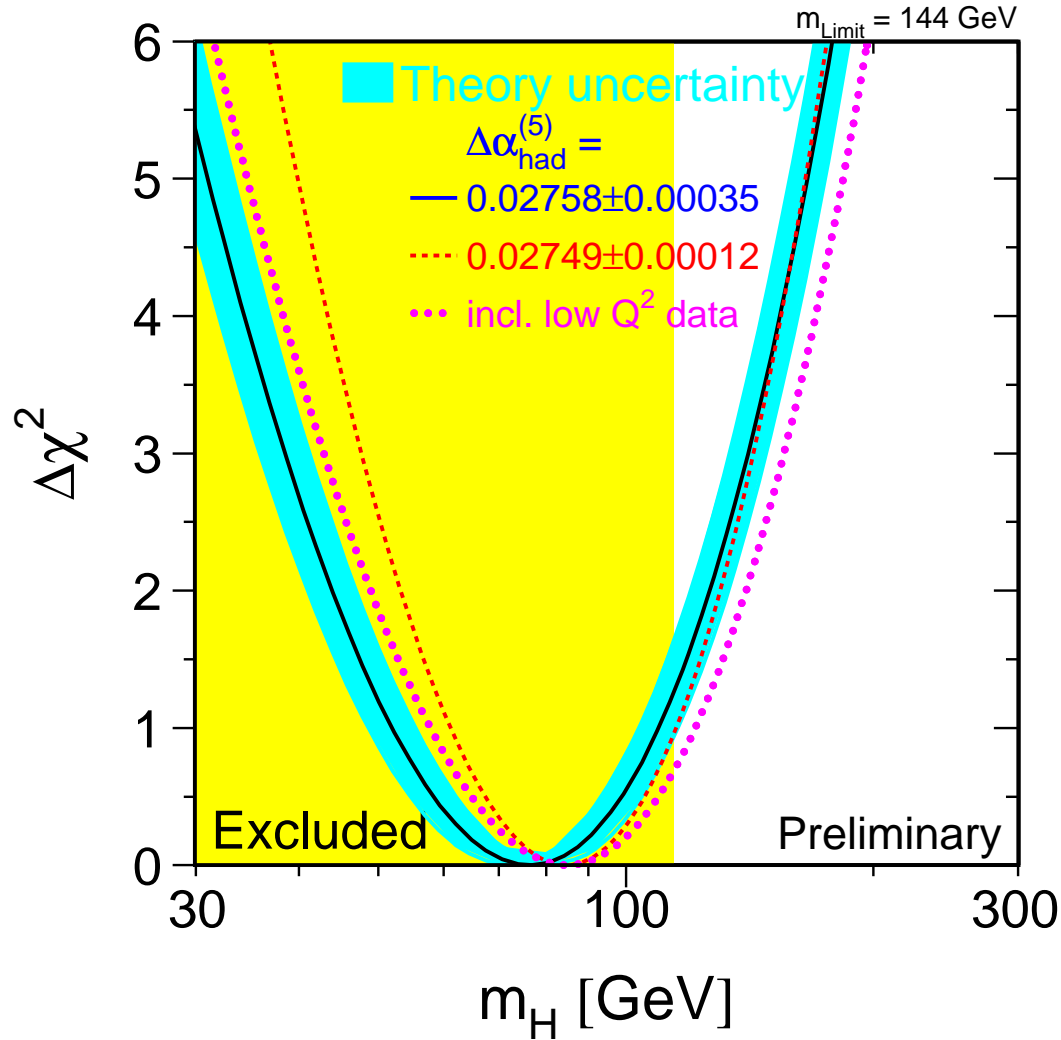
main sensitivity to m_t via



main sensitivity to M_H via



Bounds on M_H (95% C.L.)

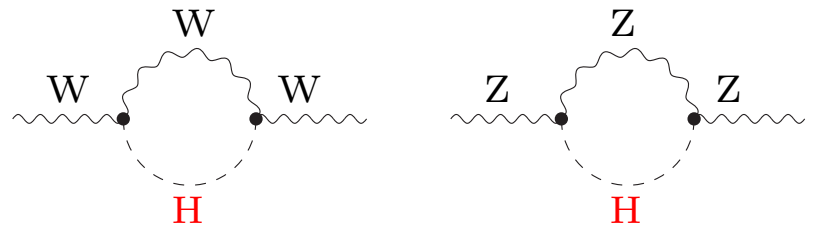


– $M_H > 114.4 \text{ GeV}$ (LEPHIGGS '02)

$e^+e^- \not\rightarrow ZH$ at LEP2

– $M_H < 144 \text{ GeV}$ (LEPEWWG '07)

fit to precision data,
i.e. via quantum corrections



Sensitivity via “high-precision observables”: $m_t, M_W, \sin^2 \theta_{\text{eff}}^{\text{lept}}$, etc.

↪ precise measurement is possible at future **ILC** !

⇒ stronger bounds on M_H

4.3 W-boson physics at LEP2

W-pair production $e^+e^- \rightarrow WW \rightarrow 4f(+\gamma)$

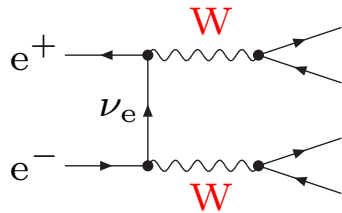


diagram dominates near W-pair threshold

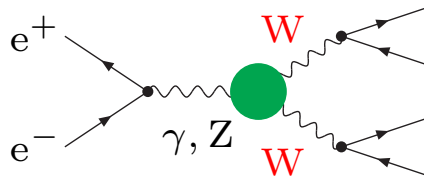


diagram contains $\gamma WW/ZWW$ couplings

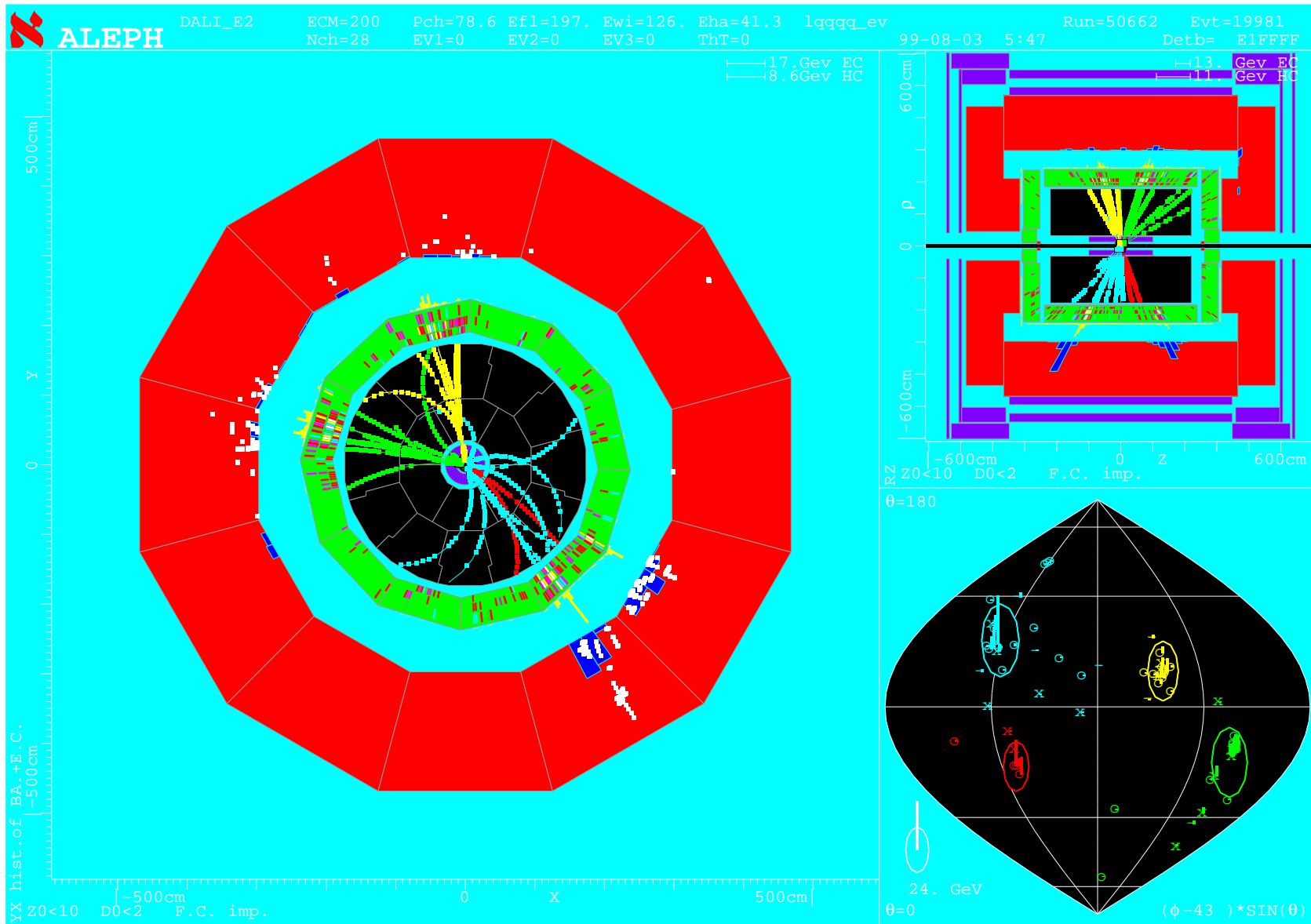
Physics issues:

- test of non-abelian structure of triple gauge-boson couplings (TGCs)
 \hookrightarrow constraint on non-standard $\gamma WW/ZWW$ couplings
- precision measurement of W-pair cross section
- precision measurement of W mass M_W
- first bounds on non-standard quartic gauge-boson couplings (QGCs)

\Rightarrow Theoretical requirement:

precise understanding of $2 \rightarrow 4$ process (0.5% level for cross section)

A typical 4-jet event observed at ALEPH

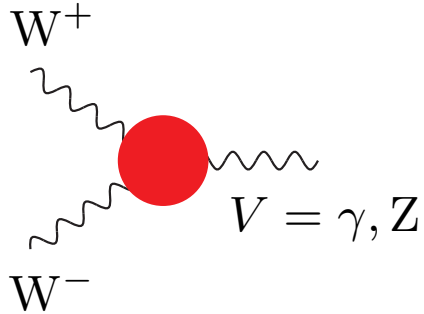


Made on 3-Aug-1999 14:42:48 by lanccon with DALI_E2.
 Filename: DC050662_019981_990803_1442.PS

(Non-)standard TGCs

Gaemers, Gounaris '79; Hagiwara, Hikasa, Peccei, Zeppenfeld '87; Bilenky, Kneur, Renard, Schildknecht '93; etc.

General parametrization (C- and P-conserving):



$$\mathcal{L}_{VWW} = -ieg_{VWW} \left\{ g_1^V (W_{\mu\nu}^+ W^{-,\mu} V^\nu - W^{-,\mu\nu} W_\mu^+ V_\nu) + \kappa_V W_\mu^+ W_\nu^- V^{\mu\nu} + \frac{\lambda_V}{M_W^2} W_{\rho\mu}^+ W_{\nu}^{-,\mu} V^{\nu\rho} \right\}$$

Meaning for static W⁺ bosons:

$$Q_W = eg_1^\gamma = \text{electric charge } (= e \text{ by charge conservation})$$
$$\mu_W = \frac{e}{2M_W} (g_1^\gamma + \kappa_\gamma + \lambda_\gamma) = \text{magnetic dipole moment}$$
$$q_W = -\frac{e}{M_W^2} (\kappa_\gamma - \lambda_\gamma) = \text{electric quadrupole moment}$$

Standard Model values:

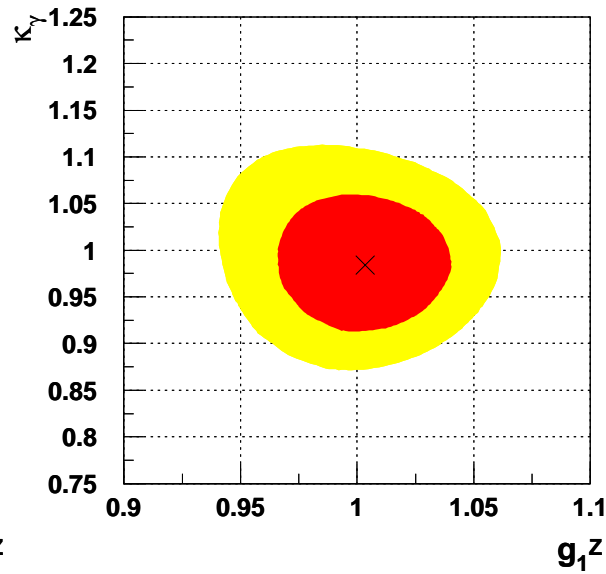
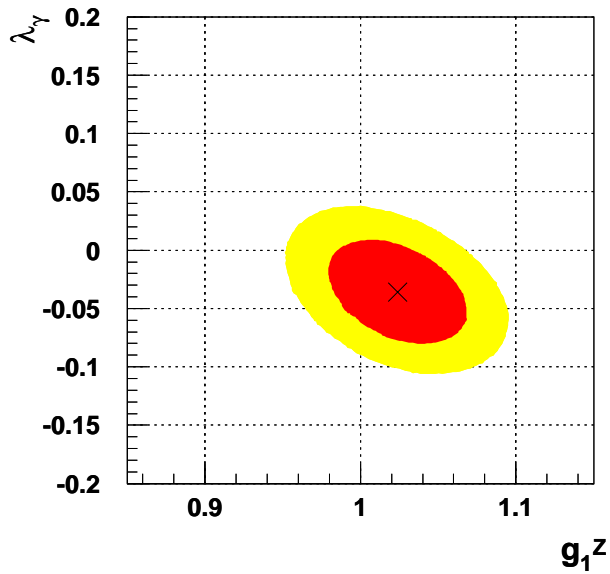
$$g_1^V = \kappa_V = 1, \quad \lambda_V = 0$$

Restriction to SU(2) × U(1)-symmetric dim-6 operators:

$$\kappa_Z = g_1^Z - (\kappa_\gamma - 1) \tan^2 \theta_W, \quad \lambda_Z = \lambda_\gamma$$

LEP2 constraints on charged TGCs

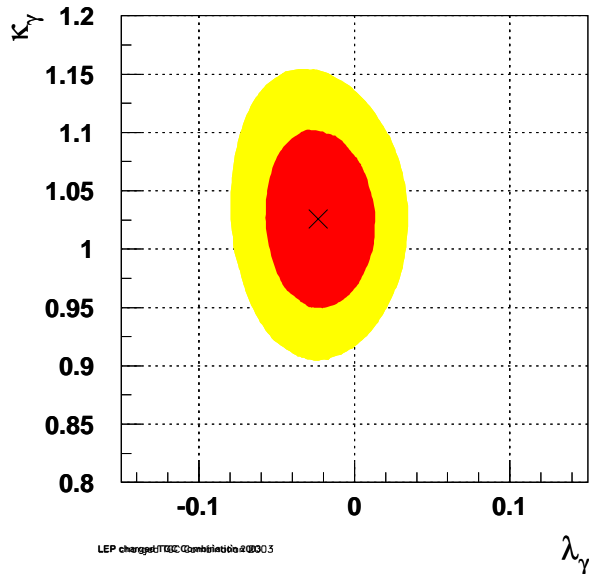
LEPEWWG '04



$$\Delta g_1^Z = -0.009^{+0.022}_{-0.021}$$

$$\Delta \kappa_\gamma = -0.016^{+0.042}_{-0.047}$$

$$\lambda_\gamma = -0.016^{+0.021}_{-0.023}$$



LEP Preliminary

- 95% c.l.
- 68% c.l.
- × 2d fit result

Standard Model values verified at the level of 2–4%

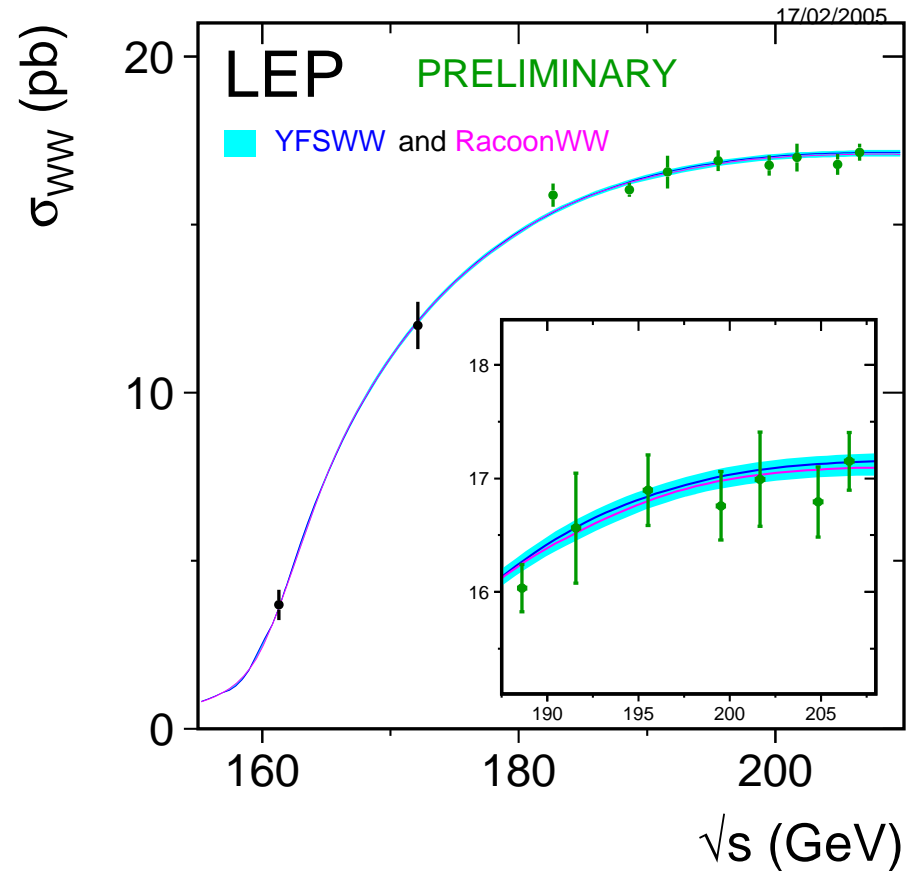
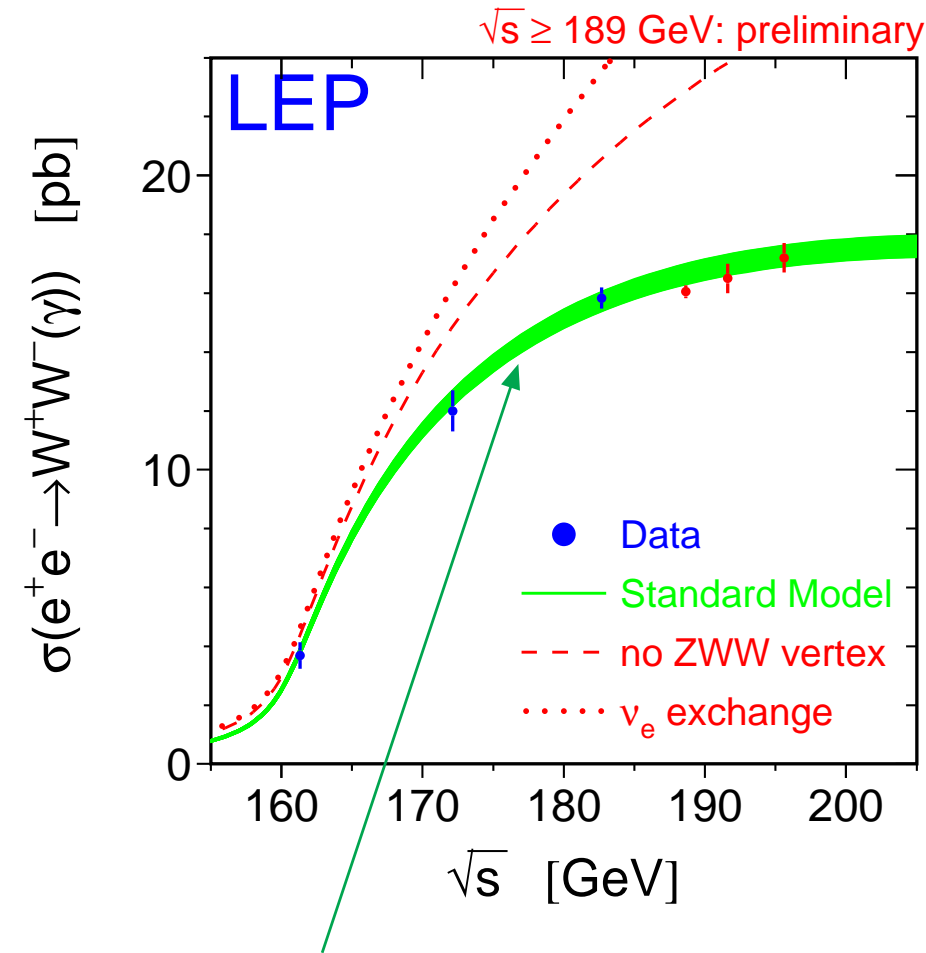
Note: TGC bounds $\sim \mathcal{O}(\text{EW corrections})$



Total WW cross section at LEP2

Status of 1999: (LEPEWWG '99)

Final (?) result: (LEPEWWG '05)



GENTLE (Bardin et al.)

only universal EW corrections

↪ theoretical uncertainty $\sim \pm 2\%$

YFSWW (Jadach et al.) / RacoonWW (Denner et al.)

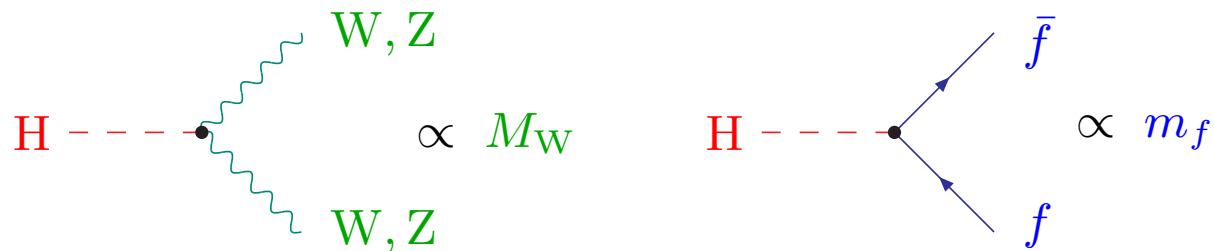
non-universal corrections included

↪ th. uncertainty $\sim \pm 0.5\%$ for $\sqrt{s} > 170$ GeV



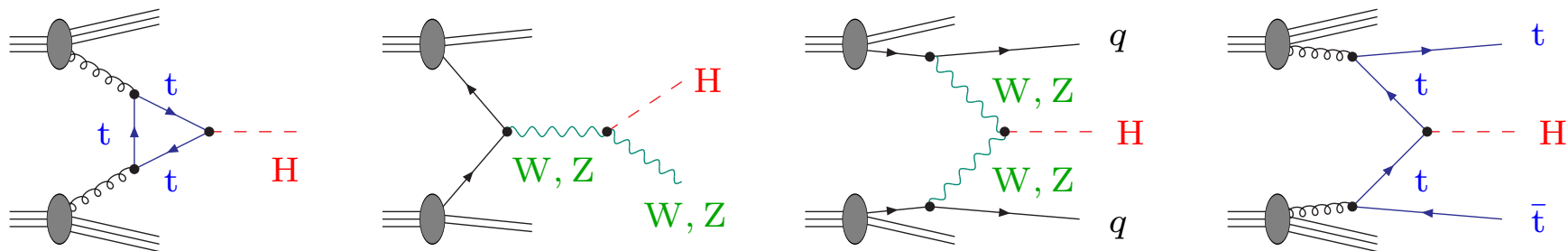
4.4 Higgs search at present and future colliders

Higgs bosons couple proportional to particle masses:

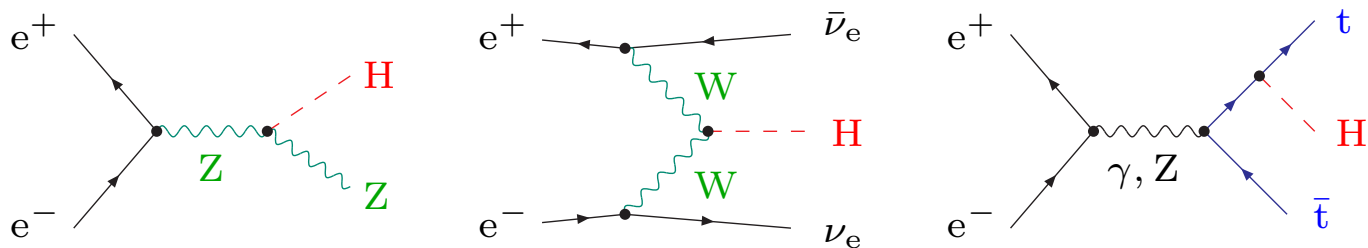


⇒ Higgs production mainly via coupling to W/Z bosons or top quarks

Processes at hadron colliders ($p\bar{p}/pp$):

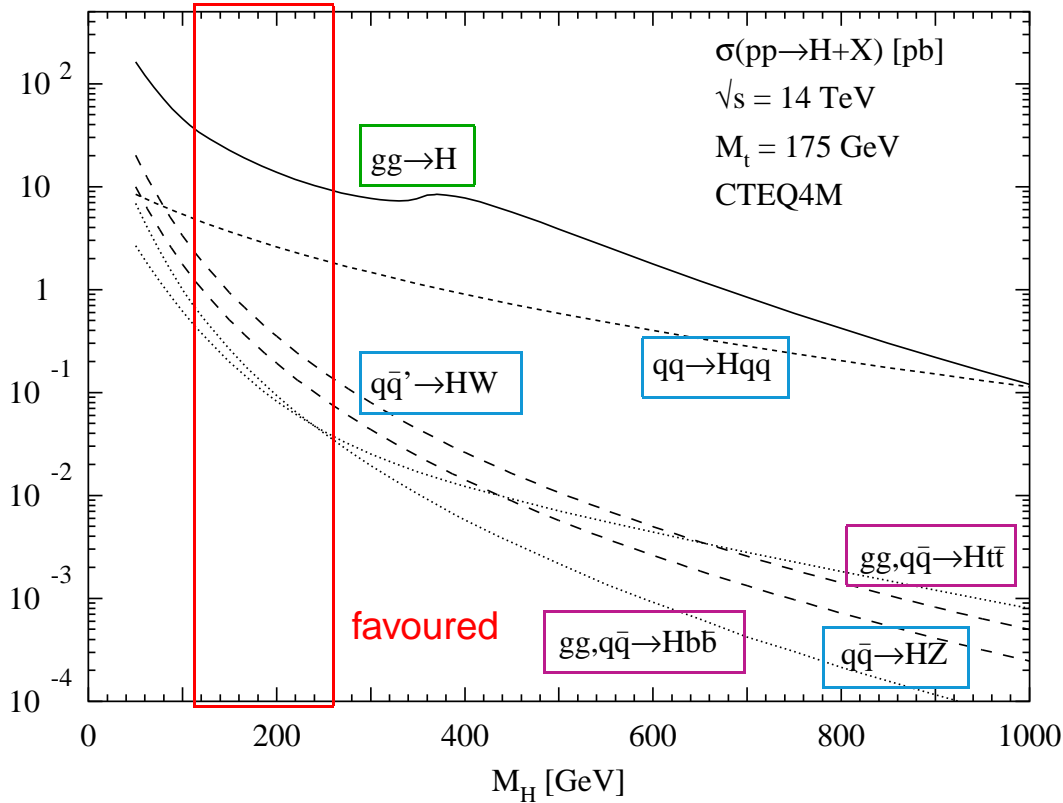


Processes at e^+e^- colliders:

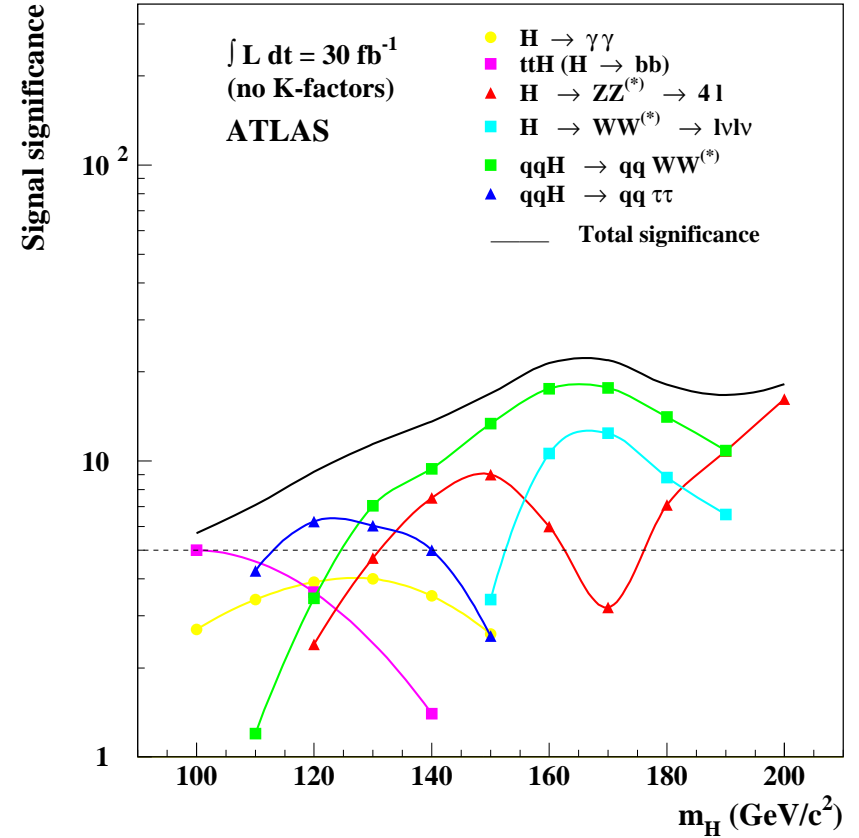


Cross sections and significance of the Higgs signal at the LHC

Spira et al. '98



ATLAS '03



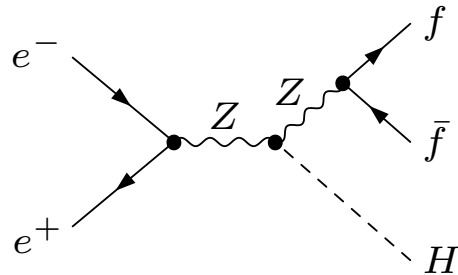
Physics goals:

- Higgs discovery, M_H measurement, decay analyses
- ratios of couplings to **W/Z bosons** and **quarks**
- extended Higgs sectors (MSSM: h, H, A, H^\pm)

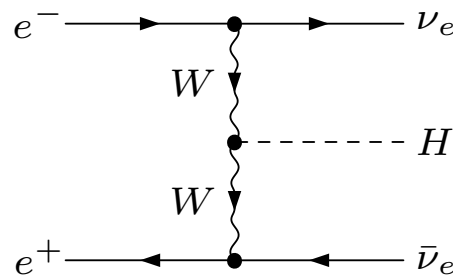


Higgs-boson production in e^+e^- annihilation

ZH production ("Higgs-strahlung")



WW fusion



WW fusion dominates

at high energies ($\sqrt{s} \gg M_H$):

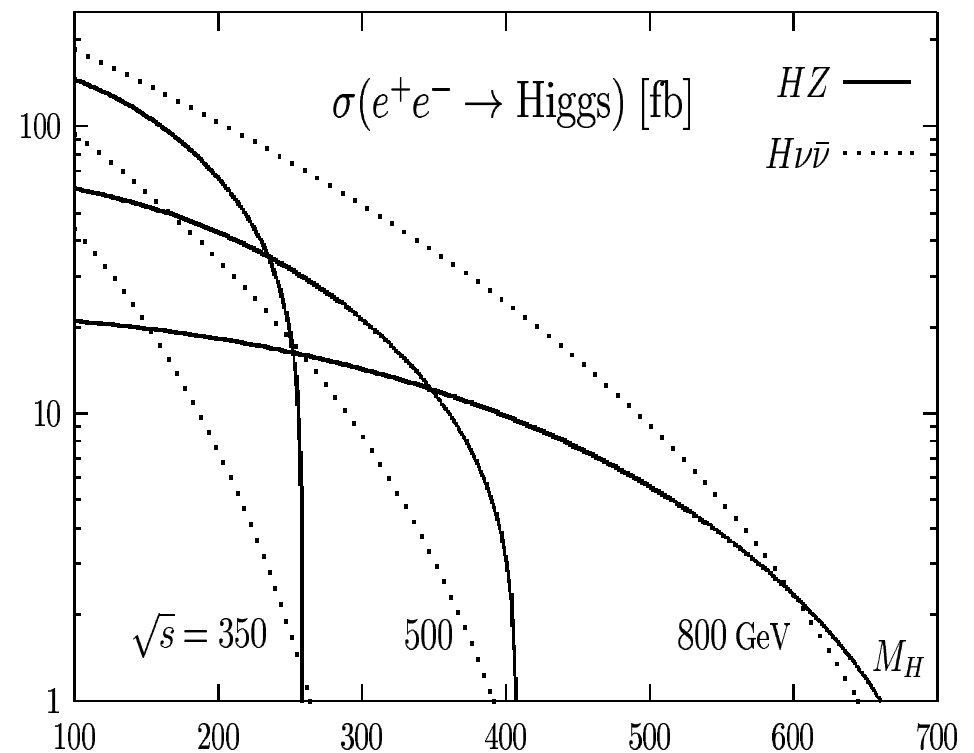
$$\sigma_{ZH} \sim \text{const} / s$$

$$\sigma_{WW} \sim \text{const} \times \ln(s/M_W^2)$$

Physics issues:

- Higgs decay width
- quantum numbers (spin, P, CP)
- measurement of couplings
- extended Higgs sectors ?

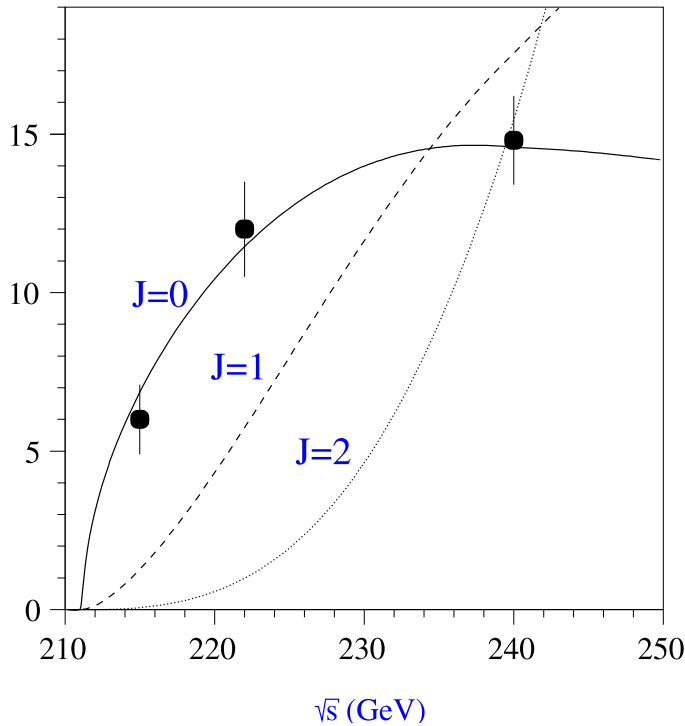
TESLA-TDR '01



Examples for Higgs studies at the ILC:

A qualitative study – spin:

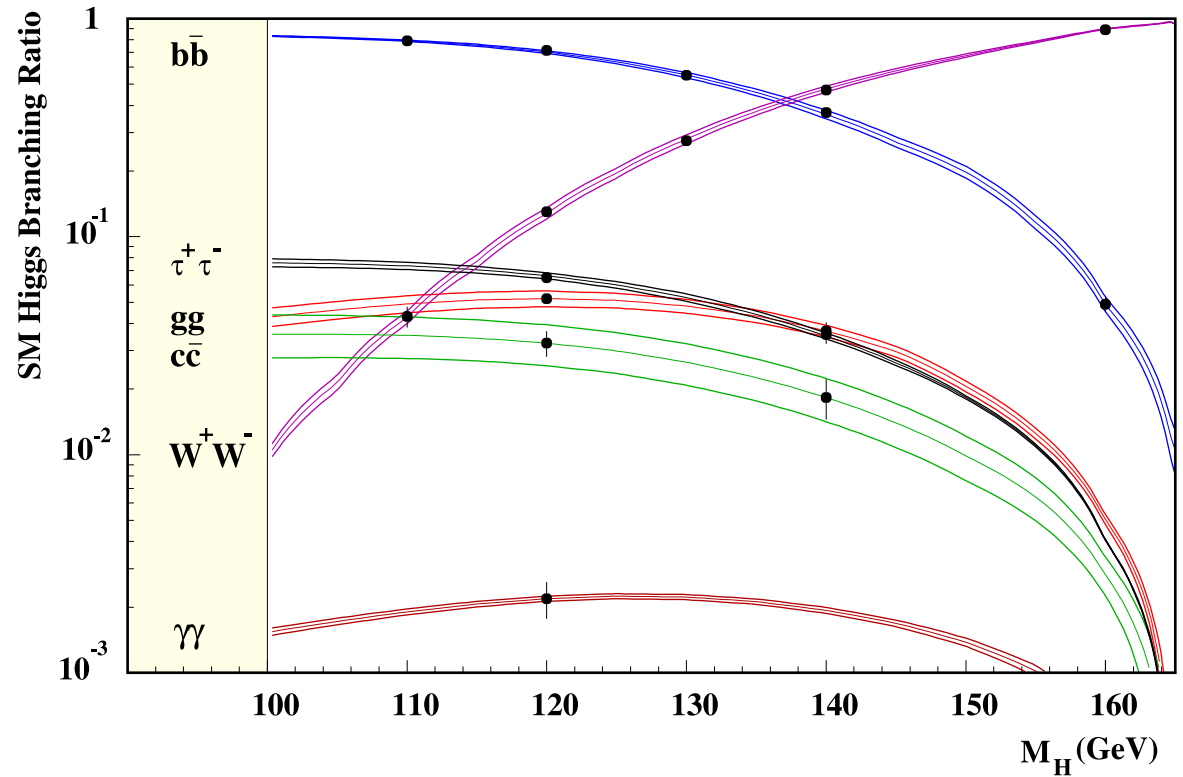
Miller et al. '01; TESLA-TDR '01



↪ spin J from rise of cross section

Precision BR measurements:

Battaglia '00; TESLA-TDR '01

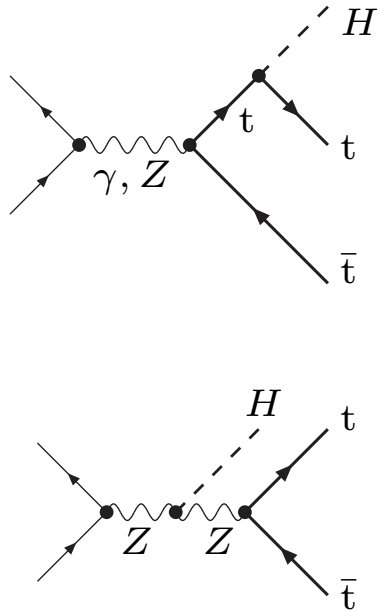


(assumed data versus theory error bands)

↪ precision test of Higgs mechanism, demarcation of SUSY Higgs bosons

Channel for analyzing the top-Yukawa coupling:

Associated Higgs production: $e^+e^- \rightarrow t\bar{t}H$

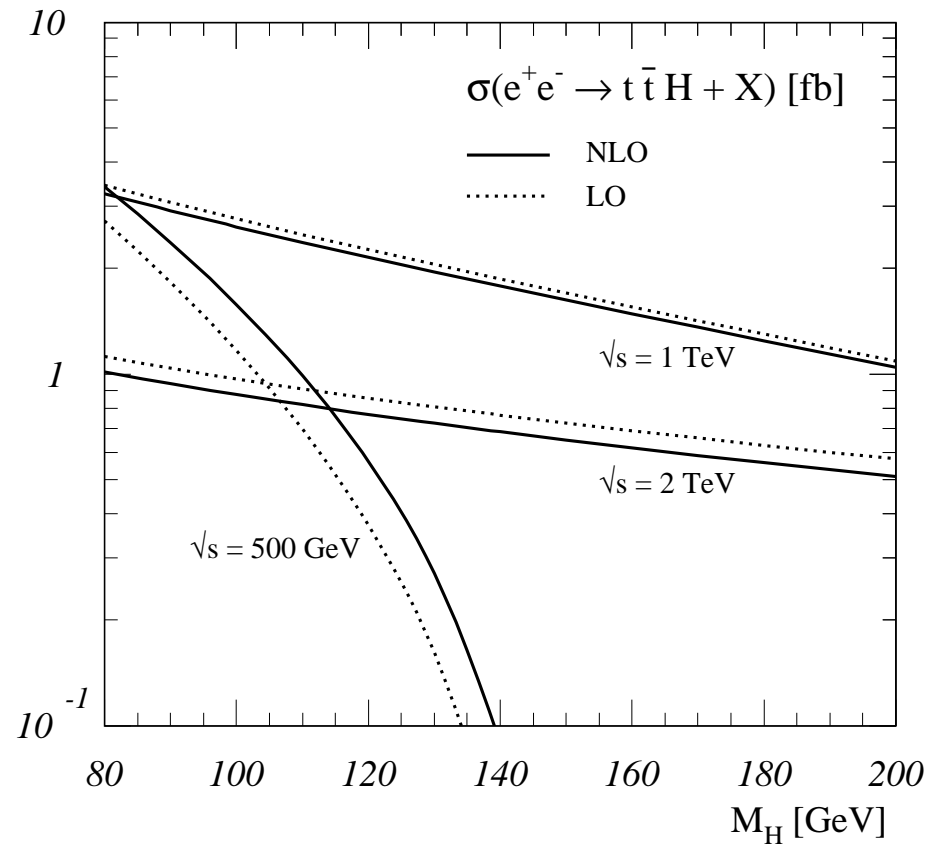


expected accuracy:

$$\Delta g_{ttH}/g_{ttH} \sim 5\%$$

QCD-corrected cross section:

Dittmaier, Krämer, Liao, Spira, Zerwas '98



4.5 The role of precision at LHC and ILC

LHC: the discovery machine (Higgs & EWSB, SUSY, etc.?)

- **QCD corrections** (at least NLO) are **substantial parts of predictions**
typical LO uncertainties \sim several 10%–100%
corrections needed for signals and many background processes
- **EW corrections also important** for many observables
(precision physics, searches at high scales, particle reconstruction, etc.)

ILC: the high-precision machine (precision \rightarrow window to higher energy)

- **old and new physics with high accuracy** (typically $\delta\sigma/\sigma \lesssim 1\%$)
 \hookrightarrow QCD and EW corrections required

- **the ultimate precision** at **GigaZ/MegaW:**

precision increases by factor ~ 10 w.r.t. LEP/SLC

$$\text{EXP: } \Delta \sin^2 \theta_{\text{eff}}^{\text{lept}} \sim 0.00001, \quad \Delta M_W \sim 7 \text{ MeV}$$

TH: go from a few 10^2 to a few 10^4 (more complicated) diagrams

\Rightarrow Precision calculations mandatory for LHC and ILC !

5 Quantum field theories and higher perturbative orders

5.1 General procedure

Formulate theory:

Lagrangian



quantization → gauge fixing, Faddeev–Popov ghosts



Perturbative evaluation:

Feynman rules



Feynman graphs



loop integrals → technical problem: **divergences (UV, IR)**



regularization → divergences mathematically meaningful



Define input parameters:

renormalization → eliminates UV divergences



Theoretical predictions:

calculation of observables (cross sections, decay widths, etc.)

↔ **IR divergences cancel for sufficiently inclusive quantities**
(e.g. inclusion of photon bremsstrahlung)

5.2 Green functions, transition amplitudes, and observables

“Amputated” Green functions $G_{\text{amp}}^{\phi_1 \dots \phi_n}$:

calculated as sum of all connected Feynman diagrams with external n legs ϕ_1, \dots, ϕ_n with external propagators (and propagator corrections) omitted

$$G_{\text{amp}}^{\phi_1 \phi_2 \phi_3} = \text{---} \bigcirc \text{---} = \text{---} \bullet \text{---} + \text{---} \bullet \text{---} \text{---} \bullet \text{---} + \text{---} \bullet \bigcirc \bullet \text{---} + \dots$$

Transition amplitude \mathcal{M}_{fi} for $|i\rangle \rightarrow |f\rangle$:

calculated from amputated Green functions $G_{\text{amp}}^{\phi_1 \dots \phi_n}$ by “LSZ reduction”:

- put external momenta to their mass shell, $p_i^2 = m_i^2$
- contract with wave functions of external particles (Dirac spinors, polarization vectors)

Note: fields must be normalized: $R_{\phi_i} = 1$ (= residue of propagator pole), otherwise multiply by $\sqrt{R_{\phi_i}}$ for each external leg

Cross section for transition $|i\rangle \rightarrow |f\rangle$:

$$\sigma = \text{flux} \times \int \text{dLIPS} |\mathcal{M}_{fi}|^2$$

“Vertex functions” $\Gamma^{\phi_1 \dots \phi_n}$ as irreducible building blocks:

- $\Gamma^{\phi_1 \phi_2} \equiv -(G^{\phi_1 \phi_2})^{-1} = -(\text{inverse propagator})$

example: scalar 2-point function

$$\Gamma^{\phi\phi}(p) = i(p^2 - m^2) + i\Sigma(p^2),$$

$\Sigma =$ self-energy = sum of 1PI graphs

$$\text{---}\bullet\text{---} = \text{---} + \text{---}\bullet\text{---}$$

1PI = 1-particle-irreducible
(graph cannot be disconnected by cutting *one* line)

$$G^{\phi\phi}(p) = \frac{i}{p^2 - m^2} + \frac{i}{p^2 - m^2} i\Sigma(p^2) \frac{i}{p^2 - m^2} + \dots \quad (\text{Dyson series})$$

$$\begin{aligned} \text{---}\circ\text{---} &= \text{---}\bullet\text{---} + \text{---}\bullet\text{---}\bullet\text{---} + \text{---}\bullet\text{---}\bullet\text{---}\bullet\text{---} + \dots \\ &= \frac{i}{p^2 - m^2 + \Sigma(p^2)} = -\left(\Gamma^{\phi\phi}(p)\right)^{-1} = -\left(\text{---}\bullet\text{---}\right)^{-1} \end{aligned}$$

- $\Gamma^{\phi_1 \dots \phi_n} \equiv G_{\text{amp}}^{\phi_1 \dots \phi_n} \Big|_{\text{only 1PI graphs}}$

example:

$$\begin{aligned} \text{---}\circ\text{---} &= \text{---}\bullet\text{---} + \text{---}\bullet\text{---}\circ\text{---}\bullet\text{---} + \text{two permutations} \\ G_{\text{amp}}^{\phi\phi\phi\phi} & \quad \Gamma^{\phi\phi\phi\phi} \quad \Gamma^{\phi\phi\phi\phi} G^{\phi\phi} \Gamma^{\phi\phi\phi\phi} \end{aligned}$$

5.3 Loop integrals and regularization

Regularization of divergences

Observation: **loop integrals involve divergences**

- **UV divergences** for $q \rightarrow \infty$, e.g.:

$$\int d^4q \frac{1}{(q^2 - m_0^2)(q^2 - m_1^2)} \sim \int \frac{dq}{q} \text{ for } q \rightarrow \infty \rightarrow \text{logarithmic divergence}$$

- **IR divergences** for $q \rightarrow q_0$, e.g.:

$$\int d^4q \frac{1}{q^2(q^2 + 2qp_1)(q^2 + 2qp_2)} \sim \int \frac{dq}{q} \text{ for } q \rightarrow 0 \rightarrow \text{logarithmic divergence}$$

“Regularization”: extension of theory by free parameter δ such that

- integrals (and thus the theory) become finite, i.e. well defined
- original theory is obtained as limiting case $\delta \rightarrow \delta_0$
 - \hookrightarrow fix input parameters x_i of regularized theory ($\delta \neq \delta_0$) by experiment
 - \Rightarrow observables must have finite limit $\delta \rightarrow \delta_0$ as functions of x_i
(independent of regularization scheme)

Convenient regularization schemes:

- **Dimensional regularization:** switch to $D \neq 4$ space-time dimensions
 - ◇ regularizes UV (and IR) divergences, respects gauge invariance, easy use
 - ◇ prescription: (μ = arbitrary reference mass, drops out in observables)

$$\int d^4q \rightarrow (2\pi\mu)^{4-D} \int d^Dq \quad \text{and } D\text{-dim. momenta, metric, Dirac algebra}$$

and analytic continuation to complex D !

- ◇ divergences appear as poles $\frac{1}{4-D}$ in results

$$\hookrightarrow \text{define } \Delta \equiv \frac{2}{4-D} - \gamma_E + \ln(4\pi) = \frac{2}{4-D} + \text{const.}$$

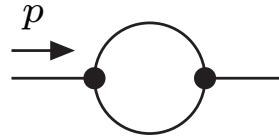
- **IR regularization by infinitesimal photon mass m_γ**
and (if relevant) by small fermion mass m_f

- ◇ prescription: photon propagator pole $\frac{1}{q^2} \rightarrow \frac{1}{q^2 - m_\gamma^2}$
- ◇ divergences appear as $\ln(m_\gamma)$ and $\ln(m_f)$ terms



Standard 1-loop integrals:

- 2-point integrals:

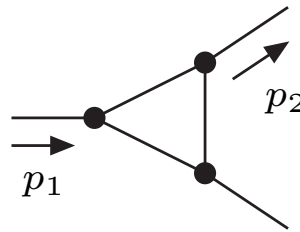


$$B_{0,\mu,\mu\nu,\dots}(p, m_0, m_1) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \frac{1, q_\mu, q_\mu q_\nu, \dots}{(q^2 - m_0^2 + i0)[(q+p)^2 - m_1^2 + i0]}$$

scalar integral $B_0 =$ logarithmically UV divergent $= \Delta +$ finite,

vector integral $B_\mu = -\frac{1}{2}p_\mu \Delta +$ finite, etc.

- 3-point integrals:



$$C_{0,\mu,\mu\nu,\dots}(p_1, p_2, m_0, m_1, m_2) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int d^D q \frac{1, q_\mu, q_\mu q_\nu, \dots}{(q^2 - m_0^2 + i0)[(q+p_1)^2 - m_1^2 + i0][(q+p_2)^2 - m_2^2 + i0]}$$

$C_0, C_\mu =$ UV finite,

$C_{\mu\nu} =$ logarithmically UV divergent $= \frac{1}{4}g_{\mu\nu} \Delta +$ finite, etc.

- 4-point integrals: $D...$ functions, etc.

Features of one-loop integrals:

- sign of infinitesimally small imaginary part $i0$ in mass terms reflects causality
- general results for 1-loop integrals known
(complicated but straightforward calculation)
 - ◇ momentum integrals can be carried out after “Feynman parametrization”
 $\hookrightarrow (n - 1)$ -dimensional integrals for n -point functions
 - ◇ B functions \rightarrow can be expressed in terms of log’s
 - ◇ C , D , etc. \rightarrow involve dilogarithms $\text{Li}_2(x) = -\int_0^x \frac{dt}{t} \ln(1 - t)$
- tensor integrals can be decomposed into Lorentz covariants:

$$B^\mu = p^\mu B_1, \quad B^{\mu\nu} = g^{\mu\nu} B_{00} + p^\mu p^\nu B_{11},$$

$$C^\mu = p_1^\mu C_1 + p_2^\mu C_2, \quad C^{\mu\nu} = p_1^\mu p_1^\nu C_{11} + p_2^\mu p_2^\nu C_{22} + (p_1^\mu p_2^\nu + p_1^\nu p_2^\mu) + g^{\mu\nu} C_{00}, \quad \text{etc.}$$

\hookrightarrow tensor coefficients B_1 , B_{ij} , C_i , etc. can be obtained as linear combinations of scalar integrals B_0 , C_0 , etc.
(e.g. by “Passarino–Veltman reduction”)

5.4 Renormalization

Propagators and 2-point functions:

Structure of one-loop self-energies (scalar case as example):

$$\Sigma(p^2) = C_1 p^2 \Delta + C_2 \Delta + \Sigma_{\text{finite}}(p^2) = \text{UV divergent}$$

Behaviour of propagator near pole for free propagation:

$$G^{\phi\phi}(p^2) = \frac{i}{p^2 - m^2 + \Sigma(p^2)} \xrightarrow{p^2 \rightarrow m^2} \frac{1}{1 + \Sigma'(m^2)} \frac{i}{p^2 - m^2 + \Sigma(m^2)}$$

↪ higher-order corrections change location and residue of propagator pole

Interaction vertices:

Example: scalar 4-point interaction $\mathcal{L}_{\phi^4} = \lambda\phi^4/4!$

$$\Gamma^{\phi\phi\phi\phi}(p_1, p_2, p_3) = i\lambda + i\Lambda(p_1, p_2, p_3)$$



momentum-dependent one-loop correction:

$$\Lambda(p_1, p_2, p_3) = C_3 \Delta + \Lambda_{\text{finite}}(p_1, p_2, p_3) = \text{UV divergent}$$

↪ higher-order corrections change coupling strengths

Structure of UV divergences:

- **Renormalizable field theories:**

UV divergences in vertex functions have analytical form of elementary vertex structures (directly related to \mathcal{L})

↪ idea: absorb divergences in free parameters

⇒ **Reparametrization of theory (=renormalization)**

Different types of renormalizable theories:

- ◇ theories with unrelated couplings of non-negative mass dimensions

 - ↪ renormalizability proven by power counting and “BPHZ procedure”

- ◇ **gauge theories** (couplings unified by gauge invariance)

 - ↪ renormalizability non-trivial consequence of gauge symmetry ‘t Hooft ’71

- **Non-renormalizable field theories:**

e.g. theories with couplings of negative mass dimensions (cf. Fermi model)

operators of higher and higher mass dimensions needed to absorb UV divergences

↪ **infinitely many free parameters**, much less predictive power

Practical procedure for renormalization:

consider original (“bare”) parameters and fields as preliminary
(denoted with subscripts “0” in the following)

↪ switch to new “renormalized” parameters and fields that obey certain conditions

Propagators and 2-point functions:

- **mass renormalization:** $m_0^2 = m^2 + \delta m^2$,
 $m^2 \stackrel{!}{=} \text{location of propagator pole} = \text{“physical mass”} \rightarrow \delta m^2 = \Sigma(m^2)$

- **wave-function ren.:** rescale fields $\phi_0 = \sqrt{Z_\phi} \phi$, $G^{\phi\phi} = Z_\phi^{-1} G^{\phi_0\phi_0}$
fix $Z_\phi = 1 + \delta Z_\phi$ such that **residue of $G^{\phi\phi}$ at $p^2 = m^2$ equals 1**
↪ $\delta Z_\phi = -\Sigma'(m^2)$

⇒ **Renormalized propagator $G^{\phi\phi}$ is UV finite:**

$$G^{\phi\phi}(p^2) = \frac{i}{p^2 - m^2 + \Sigma_{\text{ren}}(p^2)},$$

$$\begin{aligned} \Sigma_{\text{ren}}(p^2) &= \Sigma(p^2) - \Sigma(m^2) + (p^2 - m^2)\Sigma'(m^2) = \text{ren. self-energy} \\ &= \Sigma_{\text{finite}}(p^2) - \Sigma_{\text{finite}}(m^2) + (p^2 - m^2)\Sigma'_{\text{finite}}(m^2) = \text{UV finite} \end{aligned}$$

Vertex functions for interactions:

- **coupling renormalization:** $\lambda_0 = \lambda + \delta\lambda$

fix $\delta\lambda$ such that λ assumes a measured value for special kinematics p_i^{exp}

note: $\Gamma^{\phi\phi\phi\phi} = Z_\phi^2 \Gamma^{\phi_0\phi_0\phi_0\phi_0}$

$$\Leftrightarrow \delta\lambda = -2\delta Z_\phi \lambda - \Lambda(p_1^{\text{exp}}, p_2^{\text{exp}}, p_3^{\text{exp}})$$

\Rightarrow **Renormalized vertex function is UV finite:**

$$\Gamma^{\phi\phi\phi\phi}(p_1, p_2, p_3) = i\lambda + i\Lambda_{\text{ren}}(p_1, p_2, p_3),$$

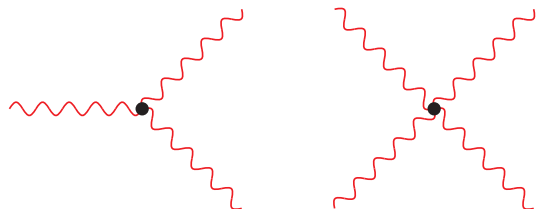
$$\Lambda_{\text{ren}}(p_1, p_2, p_3) = \Lambda_{\text{finite}}(p_1, p_2, p_3) - \Lambda_{\text{finite}}(p_1^{\text{exp}}, p_2^{\text{exp}}, p_3^{\text{exp}}) = \text{UV finite}$$

6 Electroweak Standard Model — radiative corrections

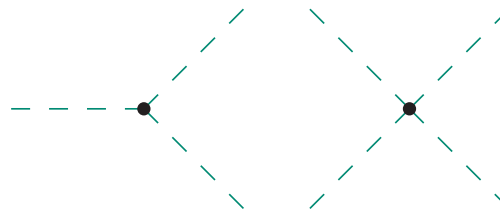
6.1 Loop corrections

Recapitulation of elementary SM couplings (vertices)

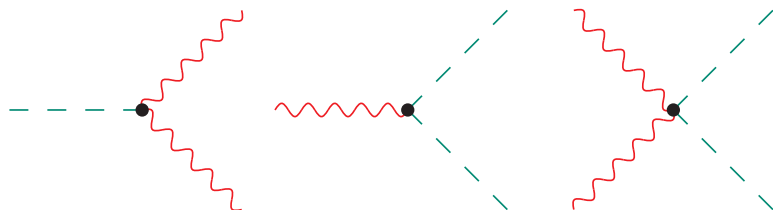
gauge-boson self-couplings:



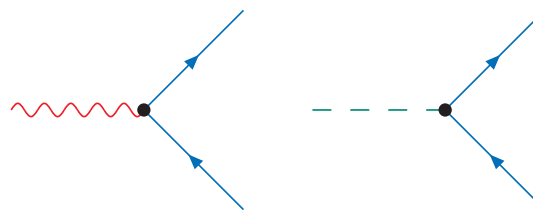
Higgs self-couplings:



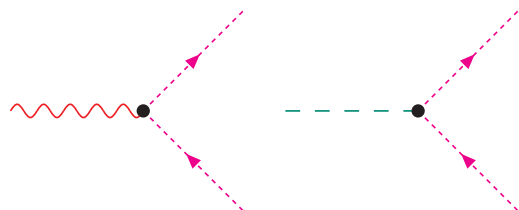
gauge-boson–Higgs couplings:



fermion couplings:



Faddeev–Popov couplings:

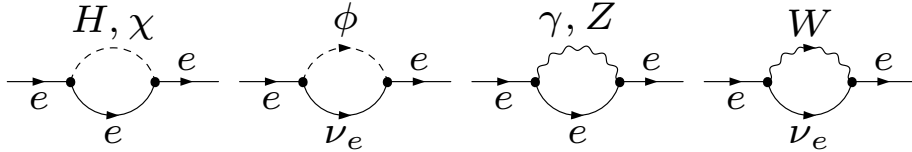


⇒ Large variety of loop diagrams !

Examples for 2-point functions at one loop: (t Hooft–Feynman gauge)

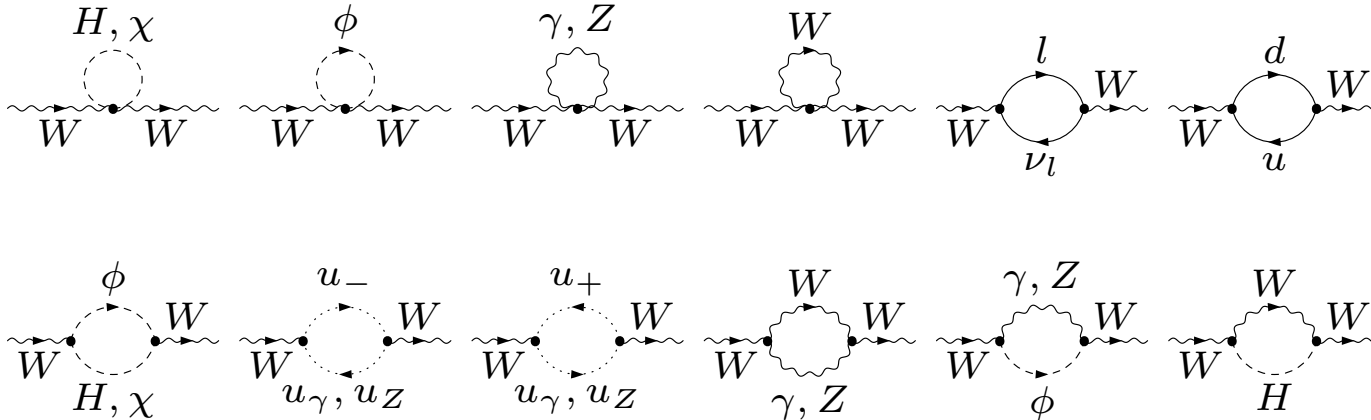
Electron self-energy:

$$\Gamma^{e\bar{e}}(p) = i(\not{p} - m_e) + i\not{p}\omega_+ \Sigma_R^e(p^2) + i\not{p}\omega_- \Sigma_L^e(p^2) + im_e \Sigma_S^e(p^2)$$



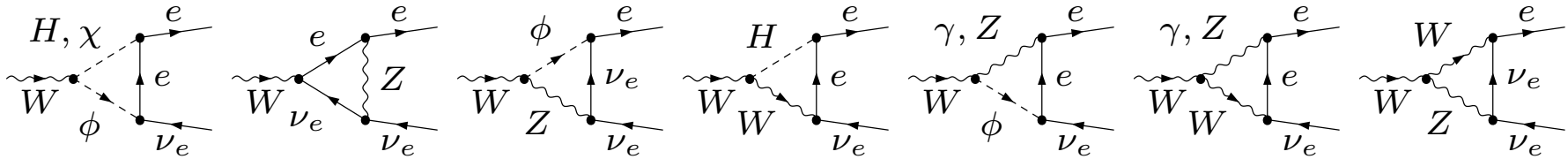
W-boson self-energy:

$$\Gamma_{\mu\nu}^{W^-W^+}(k) = -ig_{\mu\nu}(k^2 - M_W^2) - i\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \Sigma_T^W(k^2) - i\frac{k_\mu k_\nu}{k^2} \Sigma_L^W(k^2)$$

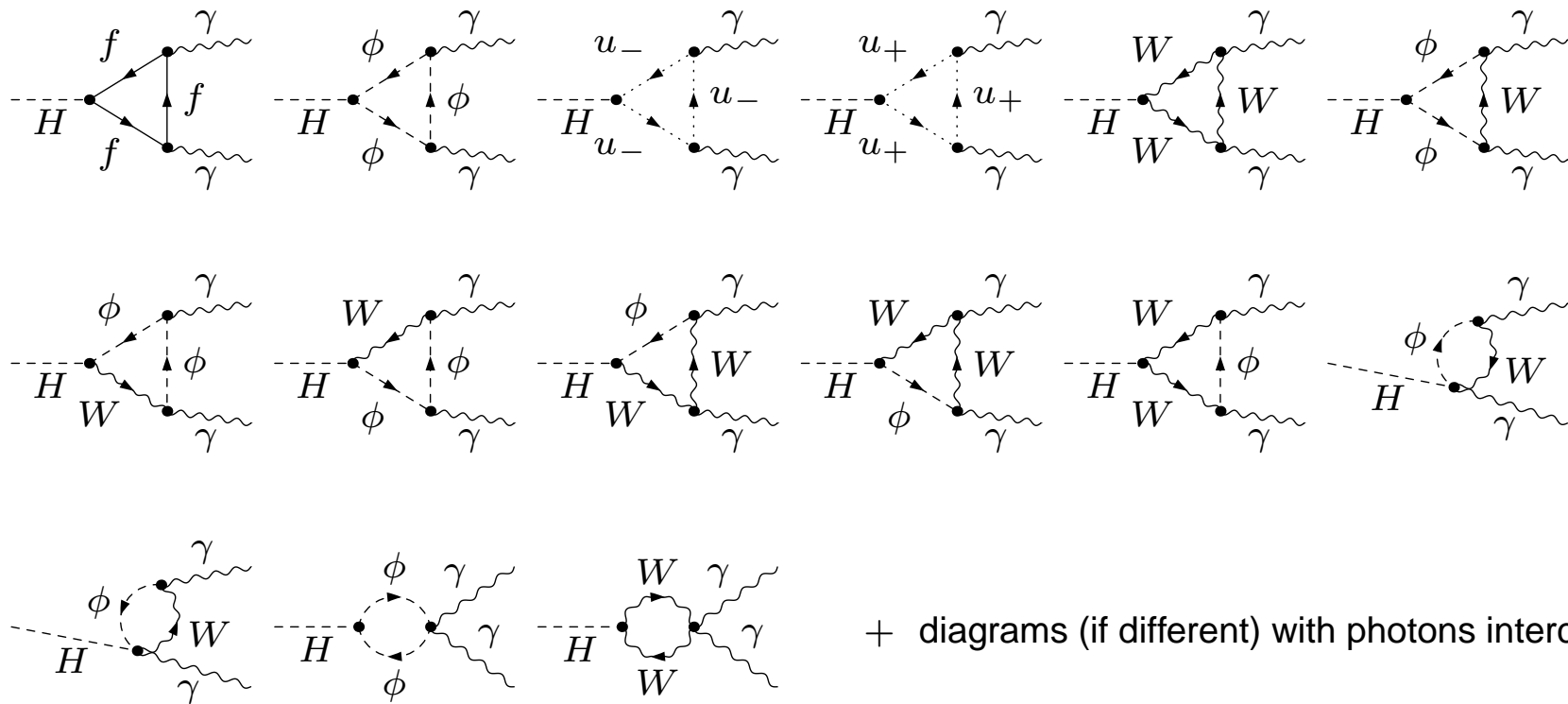


Examples for 3-point functions at one loop:

$W e \nu_e$ vertex correction:



$H\gamma\gamma$ vertex (loop induced):



6.2 Renormalization

Bare input parameters: $e_0, M_{W,0}, M_{Z,0}, M_{H,0}, m_{f,0}, V_{ij,0}$

Renormalization transformation:

- Parameter renormalization:

$$e_0 = (1 + \delta Z_e)e,$$

$$M_{W,0}^2 = M_W^2 + \delta M_W^2, \quad M_{Z,0}^2 = M_Z^2 + \delta M_Z^2, \quad M_{H,0}^2 = M_H^2 + \delta M_H^2,$$

$$m_{f,0} = m_f + \delta m_f, \quad V_{ij,0} = V_{ij} + \delta V_{ij}, \quad (\text{both } V_{ij,0}, V_{ij} \text{ unitary})$$

Note: renormalization of c_W, s_W fixed by on-shell condition $c_W = \frac{M_W}{M_Z}$
(s_W is *not* a free parameter if M_W, M_Z are used as input parameters)

- Field renormalization

$$W_0^\pm = \sqrt{Z_W} W^\pm, \quad \begin{pmatrix} Z_0 \\ A_0 \end{pmatrix} = \begin{pmatrix} \sqrt{Z_{ZZ}} & \sqrt{Z_{ZA}} \\ \sqrt{Z_{AZ}} & \sqrt{Z_{AA}} \end{pmatrix} \begin{pmatrix} Z \\ A \end{pmatrix}, \quad H_0 = \sqrt{Z_H} H,$$

$$\psi_{f,0}^L = \sqrt{Z_{ff'}^L} \psi_{f'}^L, \quad \psi_{f,0}^R = \sqrt{Z_{ff'}^R} \psi_{f'}^R$$

Note: matrix renormalization necessary to account for loop-induced mixing

Renormalization conditions:

- **Mass renormalization:**

on-shell definition: mass² is location of pole in propagator

$$\hookrightarrow \delta M_W^2 = \text{Re}\{\Sigma_T^W(M_W^2)\}, \quad \text{similar expressions for } \delta M_Z^2, \delta M_H^2, \delta m_f$$

Note: \diamond location of pole is complex for unstable particles

\hookrightarrow subtlety in all-orders definition, but not relevant at one loop
(gauge-invariant definition: mass² as real part of pole location)

\diamond other definitions of quark masses often more appropriate
(running masses, masses in effective field theories)

- **Field renormalization:** (bosons and leptons)

\diamond residues of propagators (diagonal, transverse parts) normalized to 1

$$\hookrightarrow \delta Z_W = -\text{Re}\{\Sigma_T^W{}'(M_W^2)\},$$

similar expressions for $\delta Z_{AA}, \delta Z_{ZZ}, \delta Z_H, \delta Z_{ff}^{L/R}$

\diamond suppression of mixing propagators on particle poles

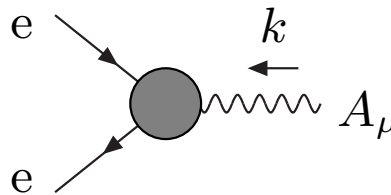
$$\hookrightarrow \text{fixes non-diagonal constants } \delta Z_{AZ}, \delta Z_{ZA}, \delta Z_{ff'}^{L/R} \quad (f \neq f')$$

Note: problems for unstable particles beyond one loop

(field-renormalization constants become complex)

Renormalization conditions: (continued)

- Charge renormalization: define e in Thomson limit



$k \rightarrow 0 \quad ie\gamma_\mu \quad \text{for on-shell electrons}$

$\Rightarrow e = \text{elementary charge of classical electrodynamics}$

$$\text{fine-structure constant } \alpha(0) = \frac{e^2}{4\pi} = 1/137.03599976$$

Gauge invariance relates δZ_e to photon wave-function renormalization:

$$\delta Z_e = -\frac{1}{2}\delta Z_{AA} - \frac{s_W}{2c_W}\delta Z_{ZA}$$

- Quark-field and CKM-matrix renormalization \rightarrow fixes $\delta Z_{qq'}^{L/R}, \delta V_{ij}$

rotation to mass eigenstates;

CKM part requires a careful (non-trivial) investigation

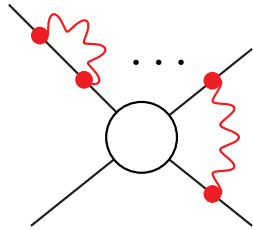
of mixing self-energies, mass eigenstates, LSZ reduction, etc.

General result: all renormalization constants can be obtained from self-energies.

6.3 IR divergences and photon bremsstrahlung

Consider processes with charged external particles, e.g., $e^+e^- \rightarrow \mu^+\mu^-$

- **Virtual corrections:** loop diagrams



IR divergences from soft virtual photons ($q \rightarrow 0$)

$$\int \frac{d^4 q \dots}{(q^2 - m_\gamma^2)(2qp_1)(2qp_2)} \rightarrow C \ln(m_\gamma)$$

- **“Real” corrections:** photon bremsstrahlung

$$\int \frac{d^3 \mathbf{q}}{2q_0} \left| \text{diagram} \right|^2$$

IR divergences from soft real photons ($\mathbf{q} \rightarrow 0$)

$$\int \frac{d^3 \mathbf{q} \dots}{\sqrt{\mathbf{q}^2 + m_\gamma^2}(2qp_1)(2qp_2)} \rightarrow -C \ln(m_\gamma)$$

Bloch–Nordsieck theorem:

IR divergences of virtual and real corrections cancel in the sum

↪ virtual and soft-photon corrections cannot be discussed separately

↔ related to limited experimental resolution of soft photons

⇒ Cross-section predictions necessarily depend on treatment of photon emission (energy and angular cuts)

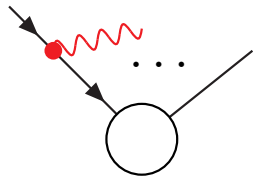
Separation of soft and hard photons:

Why? cancellation of $\ln(m_\gamma)$ terms delicate in practice, but terms are universal

- **soft photons**, $m_\gamma < E_\gamma < \Delta E \ll Q =$ typical scale of the process
 \hookrightarrow correction is universal factor δ_{soft} to Born cross section
 relatively simple analytical expression with explicit $C \ln(\Delta E/m_\gamma)$ terms
- **hard photons**, $E_\gamma > \Delta E$
 \hookrightarrow Monte Carlo integration of full radiative process, but with $m_\gamma = 0$
 $-C \ln(\Delta E)$ terms emerge numerically

$\ln(\Delta E)$ contributions cancel numerically in sum for small ΔE up to $\mathcal{O}(\Delta E/E)$

Calculation of soft-photon factor:



$$= A(p - q) \frac{i(\not{p} - \not{q} + m_f)}{(p - q)^2 - m_f^2} (iQ_f e) \not{\epsilon}_\gamma^* u_f(p)$$

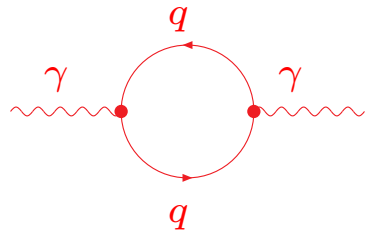
$$\underset{q \rightarrow 0}{\sim} -Q_f e \frac{\epsilon_\gamma^* p}{qp} A(p) u_f(p) = -Q_f e \frac{\epsilon_\gamma^* p}{qp} \mathcal{M}_{\text{Born}}$$

“Eikonal factorization” holds for all charged particles (spin 0, $\frac{1}{2}$, 1)

$$\Rightarrow \delta_{\text{soft}} = -\frac{\alpha}{2\pi^2} \int_{m_\gamma < q_0 < \Delta E} \frac{d^3 \mathbf{q}}{2q_0} \sum_{i,j} \frac{(\pm Q_i)(\pm Q_j)(p_i p_j)}{(qp_i)(qp_j)} \quad (i = \text{particle with charge } Q_i \text{ incoming}(+) \text{ or outgoing } (-))$$

6.4 The universal radiative corrections $\Delta\alpha$ and $\Delta\rho$

Running electromagnetic coupling $\alpha(s)$:



becomes sensitive to unphysical quark masses m_q
for $|s|$ in GeV range and below (non-perturbative regime)

\hookrightarrow charge-renormalization constant δZ_e sensitive to m_q

Solution: fit hadronic part of $\Delta\alpha(s) = -\text{Re}\{\Sigma_{\text{T,ren}}^{AA}(s)/s\}$ and thus of δZ_e

via dispersion relations to $R(s) = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$

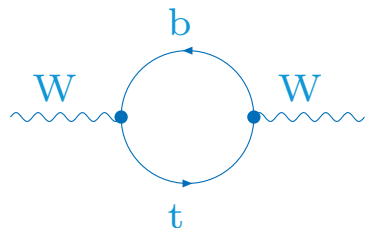
Jegerlehner et al.

\Rightarrow Running elmg. coupling: $\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha_{\text{ferm} \neq \text{top}}(s)}$

Leading correction to the ρ -parameter:

mass differences in fermion doublets break custodial SU(2) symmetry

\hookrightarrow large effects from bottom–top loops in W self-energy Veltman '77



$$\Delta\rho_{\text{top}} \sim \frac{\Sigma_{\text{T}}^{ZZ}(0)}{M_Z^2} - \frac{\Sigma_{\text{T}}^{WW}(0)}{M_W^2} \sim \frac{3G_\mu m_t^2}{8\sqrt{2}\pi^2}$$

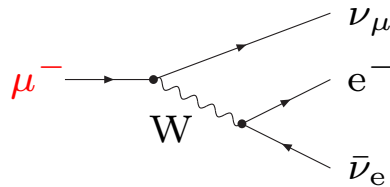
Precision calculation of M_W via μ decay

$\hookrightarrow M_W$ as function of $\alpha(0)$, G_μ , M_Z and the quantity Δr

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi\alpha(0)}{\sqrt{2}G_\mu} (1 + \Delta r)$$

Δr comprises quantum corrections to μ decay
(beyond electromagnetic corrections in Fermi model)

Lowest order:



$\mathcal{O}(\alpha)$ corrections:

$$\Delta r_{1\text{-loop}} = \Delta\alpha(M_Z^2) - \frac{c_W^2}{s_W^2} \Delta\rho_{\text{top}} + \Delta r_{\text{rem}}(M_H)$$

Sirlin '80, Marciano, Sirlin '80

$\sim 6\%$

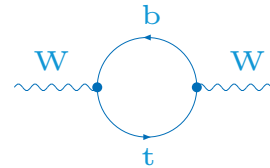
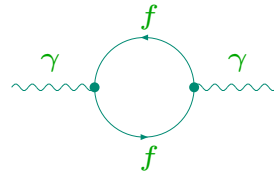
$\sim 3\%$

$\sim 1\%$

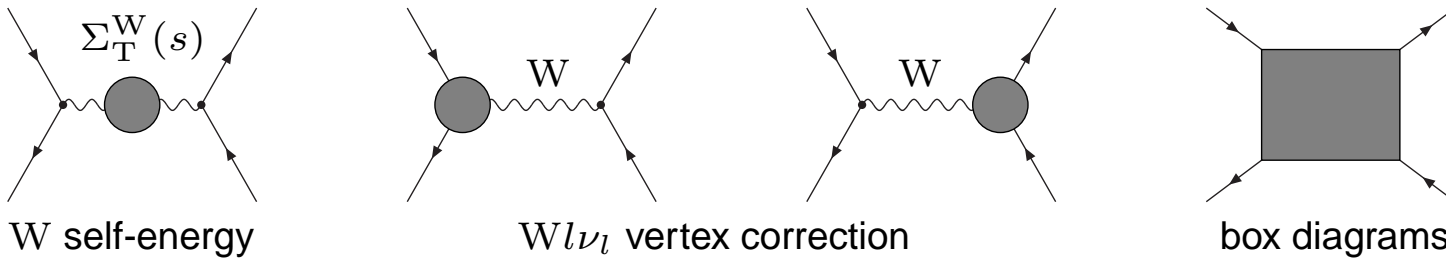
$$\alpha \ln(m_f/M_Z)$$

$$G_\mu m_t^2$$

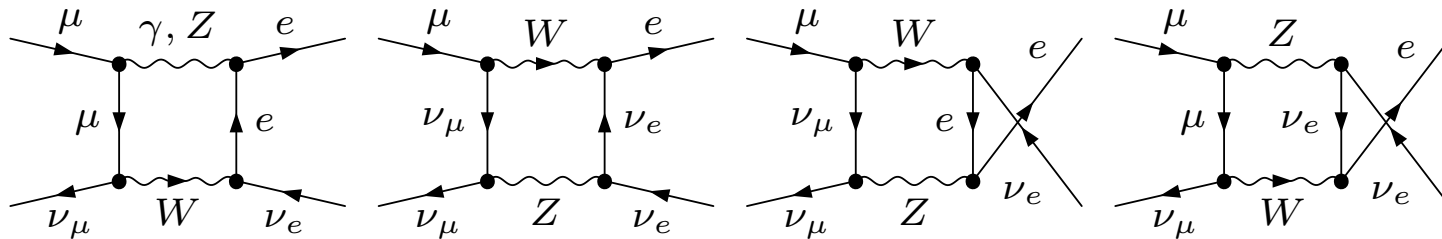
$$\alpha \ln(M_H/M_Z)$$



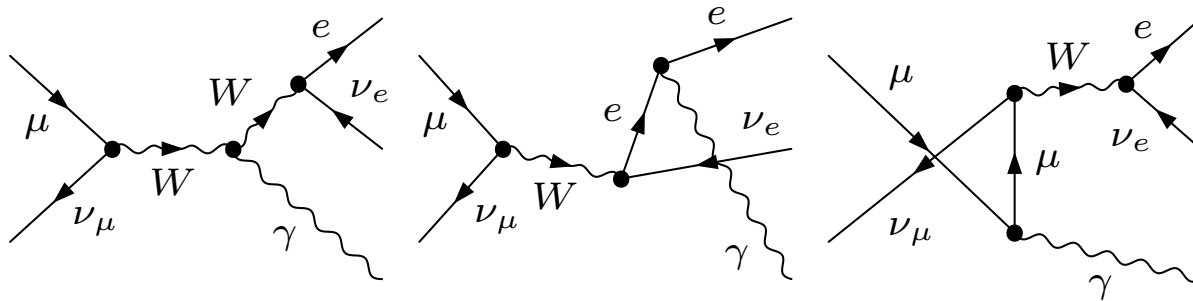
Virtual correction – 1-loop diagrams:



e.g.:

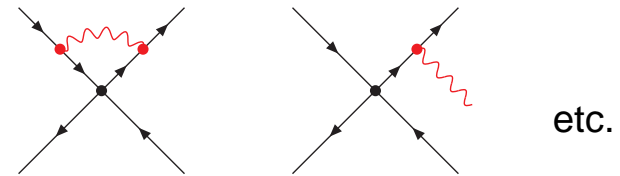


Real correction – 1-photon bremsstrahlung:

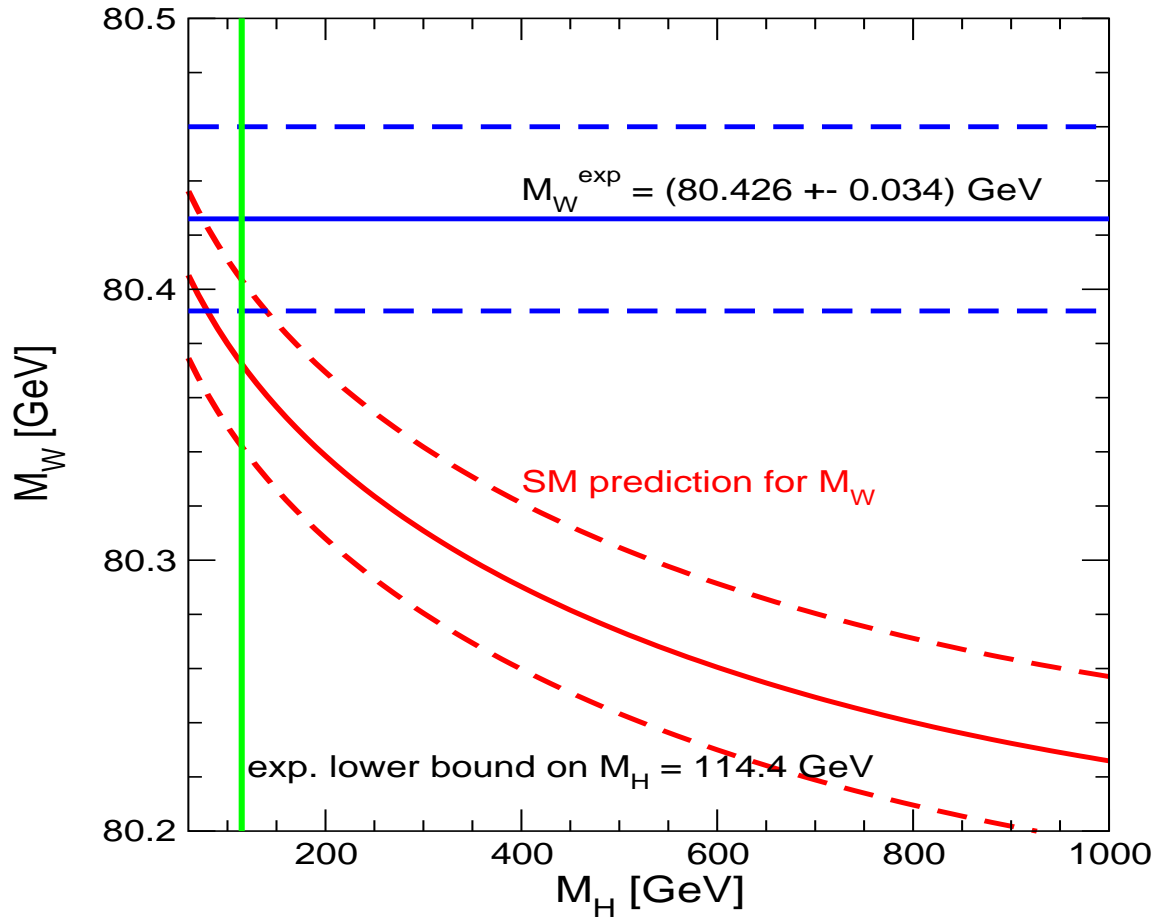


Consistent use of G_μ :

Photonic QED corrections are treated in the Fermi model and subtracted from Δr



State-of-the-art prediction of M_W from muon decay:



Hollik et al. '03

Theoretical uncertainty:

status '00: $\Delta M_W \sim 6 \text{ MeV}$

status '06: $\Delta M_W \sim 4 \text{ MeV}$

Experimental error:

status '06: $\Delta M_W \sim 29 \text{ MeV}$

ILC(?): $\Delta M_W \sim 7 \text{ MeV}$

Prediction includes:

- full electroweak corrections of $\mathcal{O}(\alpha)$ (1-loop level)
- full electroweak corrections of $\mathcal{O}(\alpha^2)$ (2-loop level)
 (v.Ritbergen,Stuart '98; Seidensticker,Steinhauser '99;
 Freitas,Hollik,Walter,Weiglein '00-'02; Awramik,Czakon '02/'03; Onishchenko,Veretin '02)
- various improvements by universal corrections to ρ -parameter

Literature

- Textbooks:
 - ◇ Böhm/Denner/Joos: “Gauge Theories of the Strong and Electroweak Interaction”
 - ◇ Cheng/Li: “Gauge Theory of Elementary Particle Physics”
 - ◇ Collins: “Renormalization”
 - ◇ Ellis/Stirling/Webber: “QCD and Collider Physics”
 - ◇ Itzykson/Zuber: “Quantum Field Theory”
 - ◇ Peskin/Schroeder: “An Introduction to Quantum Field Theory”
 - ◇ Weinberg: “The Quantum Theory of Fields, Vol. 1: Foundations”;
“The Quantum Theory of Fields, Vol. 2: Modern Applications”
- Some reviews on dedicated topics:
 - ◇ Z-boson production at LEP1/SLC:
“Z Physics at LEP1”, eds. G. Altarelli, R. Kleiss and C. Verzegnassi (CERN 89-08), Vol. 1;
“Reports of the Working Group on Precision Calculations for the Z Resonance”, eds. D. Bardin,
W. Hollik and G. Passarino (CERN 95-03);
D.Y. Bardin, M. Grünewald and G. Passarino, hep-ph/9902452.
 - ◇ W-pair production at LEP2:
W. Beenakker *et al.*, in “Physics at LEP2”, eds. G. Altarelli, T. Sjöstrand and F. Zwirner (CERN 96-01,
Geneva, 1996), Vol. 1, p. 79 [hep-ph/9602351];
M. W. Grünewald *et al.*, in “Reports of the Working Groups on Precision Calculations for LEP2
Physics”, eds. S. Jadach, G. Passarino and R. Pittau (CERN 2000-009), p. 1 [hep-ph/0005309].
 - ◇ SM Higgs physics:
A. Djouadi, hep-ph/0503172 and references therein



Literature (continued)

- Experimental results widely taken from:
 - ◇ LEPEWWG: <http://lepewwg.web.cern.ch/LEPEWWG/>
 - ◇ LEPHiggs: <http://lephiggs.web.cern.ch/LEPHIGGS/www/Welcome.html>
- (Incomplete) list of articles on techniques for radiative corrections:
 - ◇ one-loop integrals:
 - G. 't Hooft and M. Veltman, Nucl. Phys. B 153 (1979) 365;
 - G. Passarino and M. Veltman, Nucl. Phys. B 160 (1979) 151;
 - W. Beenakker and A. Denner, Nucl. Phys. B 338 (1990) 349;
 - A. Denner, U. Nierste and R. Scharf, Nucl. Phys. B 367 (1991) 637;
 - A. Denner and S. Dittmaier, Nucl. Phys. B 734 (2006) 62 and references therein
 - ◇ renormalization of the electroweak SM:
 - K. I. Aoki, Z. Hioki, M. Konuma, R. Kawabe and T. Muta, Prog. Theor. Phys. Suppl. 73 (1982) 1;
 - M. Böhm, W. Hollik and H. Spiesberger, Fortsch. Phys. 34 (1986) 687;
 - W. F. Hollik, Fortsch. Phys. 38 (1990) 165;
 - A. Denner, Fortsch. Phys. 41 (1993) 307;
 - A. Denner, S. Dittmaier and G. Weiglein, Nucl. Phys. B 440 (1995) 95
 - ◇ IR structure of photon radiation:
 - D. R. Yennie, S. C. Frautschi and H. Suura, Annals Phys. 13 (1961) 379.

