Constraining CP violation in neutral meson mixing with theory input

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Work in collaboration with M. Freytsis and Z. Ligeti [1203.3545 hep-ph]
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1. **Introduction**
   - Motivation
   - Description of Meson Oscillation
   - Theoretical Predictions of Oscillation Parameters

2. **Theoretical Constraints on the Mixing Parameters**
   - Unitarity Constraint
   - Deriving a Relation using Theoretical Input
   - Application to Recent Data

3. **Summary**
   - Discussion
   - Summary
Motivation

- The Standard Model has passed all precision tests
  1. CERN: Z discovery, test of the gauge structure
  2. Flavour factories: Test of the flavour sector
  3. Tevatron: Discoveries, top, $B_s - \bar{B}_s$ oscillation, ...
  4. LHC: Up to now no significant new discoveries
- Only a few tensions $\sim 2 - 3\sigma$
- Most hints for New Physics in flavour physics sector

Promising Channels: Flavour changing neutral currents (FCNC)
- Forbidden at tree level $\Rightarrow$ NP can enter at the same order
- $\Delta F = 1$ processes: Rare decays
- $\Delta F = 2$ processes: Meson oscillation / mixing
- Focus here on $M - \bar{M}$ oscillation (especially $B_{d/s} - \bar{B}_{d/s}$)
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**Mixing and CP Violation: Origin and Consequences**

**CKM Matrix**
- Diagonalize up- and down-type quark mass matrices simultaneously
  \[ V_{\text{ckm}} = V^{(u)} W^{(d)\dagger} \]
- 3 generations \( \Rightarrow \) \( V_{\text{ckm}} \) has 3 angles and 1 complex phase

**Consequences**
- CP violation if all masses are non-degenerate
- Transitions between different generations
  \( \Rightarrow \) Flavor changing neutral currents at the loop-level
CKM Matrix

- Diagonalize up- and down-type quark mass matrices simultaneously
- Missmatch in charged current described by CKM matrix
  \[ V_{\text{ckm}} = V^{(u)} W^{(d)\dagger} \]
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  \[ V_{\text{CKM}}^\dagger V_{\text{CKM}} = \begin{pmatrix} V_{ud}^* & V_{cd}^* & V_{td}^* \\ V_{us}^* & V_{cs}^* & V_{ts}^* \\ V_{ub}^* & V_{cb}^* & V_{tb}^* \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]
Introduction
Theoretical Constraints on the Mixing Parameters
Motivation
Description of Meson Oscillation
Theoretical Predictions of Oscillation Parameters

The CKM Matrix

[CKMfitter: http://ckmfitter.in2p3.fr/]

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CP Violating in Mixing

General Comments

- Occurs in $P_0 \leftrightarrow \bar{P}_0$ oscillations
- Flavor specific final states
  \[ P_0 \to f \leftrightarrow \bar{P}_0 \]
- Necessary condition: $\left|\frac{q}{p}\right| \neq 1$
- Rates for $B$ and $\bar{B}$ differ

Example of Process

Semi-leptonic asymmetry: $P_0 \to X\ell^+\bar{\nu}_\ell$ and $\bar{P}_0 \to X\ell^-\nu_\ell$

\[ A_{sl} \equiv \frac{\Gamma(P_0 \to X\ell^-) - \Gamma(\bar{P}_0 \to X\ell^+)}{\Gamma(P_0 \to X\ell^-) + \Gamma(\bar{P}_0 \to X\ell^+)} = \frac{1 - |q/p|^4}{1 + |q/p|^4} \]
Description of Neutral Meson Mixing

- Two state system with interplay of oscillation and decay
- Mass matrix $M$ and decay width matrix $\Gamma$ are hermitian

\[
i \frac{\partial}{\partial t} \left( |P_0\rangle - |\bar{P}_0\rangle \right) = \left[ \begin{pmatrix} M_{11} & M_{12} \\ M_{12}^* & M_{11} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_{11} \end{pmatrix} \right] \left( |P_0\rangle - |\bar{P}_0\rangle \right)
\]

Diagonalization

- Solution for mass eigenstates

\[
|P_{H,L}\rangle = \frac{p |P^0\rangle \mp q |\bar{P}_0\rangle}{\sqrt{|p|^2 + |q|^2}}, \quad \frac{q^2}{p^2} = \frac{2M_{12}^* - i\Gamma_{12}^*}{2M_{12} - i\Gamma_{12}}
\]

- Mass eigenstates do not need to coincide with CP eigenstates

\[
\delta \equiv \langle P_H | P_L \rangle = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} = 1 - \frac{|q/p|^2}{1 + |q/p|^2} = \frac{1 - \sqrt{1 - A_{sl}^2}}{A_{sl}} \approx \frac{1}{2} A_{sl}
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Description of Neutral Meson Mixing

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Test of the Standard Model

Need to predict three parameters to compare with SM

\[
\Delta M = 2 \text{Re} \sqrt{(M_{12} - i/2\Gamma_{12})(M^*_{12} - i/2\Gamma^*_{12})} \approx 2 |M_{12}|
\]

\[
\Delta \Gamma = -4 \text{Im} \sqrt{(M_{12} - i/2\Gamma_{12})(M^*_{12} - i/2\Gamma^*_{12})} \approx 2 |\Gamma_{12}| \cos[\text{Arg}(-\Gamma_{12}/M_{12})]
\]

\[
\delta = (1 - |q/p|^2)/(1 + |q/p|^2) \approx 1/2 \text{Im} \Gamma_{12}/M_{12}
\]

Mixing Parameter Input

- \(M_{12}\): Dominated by dispersive part of \(\Delta B = 2\) operator
- \(\Gamma_{12}\): Dominated by absorptive part of \(\Delta B = 1\) op. double insertion
- Main theoretical uncertainties
  1. Operator product expansion in physical region
  2. Expansion in small energy release \(m_b - 2m_c < 2 \text{ GeV}\)

\[\text{[1008.1593,1203.0238]}\]
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Effective Theory at the scale of the $B$

\[ \mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} \lambda_{\text{CKM}} \sum_i C_i(\mu)O_i(\mu) \]

- Current-current operators
- Electroweak/QCD Penguins
- Magnetic Penguins
- Semi-leptonic operators
- $\Delta F = 2$ operators

Allows for Systematic Calculation: Heavy Quark Expansion (HQE)

- Perturbative $\alpha_s$ corrections
- Non-perturbative $1/m_{b,c}$ corrections
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The Hamilton Matrix: Computing Mixing Parameters

- Matrix to be understood in \( M_1 \equiv M \rightarrow \tilde{M} \equiv M_2 \) space
- Weak interaction sets scale: “Wigner-Weisskopf” approximation
  \[ \Rightarrow \text{Expansion in powers of } G_F \triangleq \text{Number of } \mathcal{H}_{\text{weak}} \text{ Insertions} \]
- Use rest-frame of the meson
  \[ \left[ \mathcal{M} - \frac{i}{2} \Gamma \right]_{ij} = M_M \delta^{(0)}_{ij} + \frac{1}{2 M_M} \sum_n \frac{\langle M_i | \mathcal{H}_{\text{weak}} | n \rangle \langle n | \mathcal{H}_{\text{weak}} | M_j \rangle}{M_M^{(0)} - E_n + i \epsilon} + \ldots \]
- Sum includes phase-space of final state
- Decompose into dispersive and absorptive part “optical theorem”
  \[ \frac{1}{\omega + i \epsilon} = \mathcal{P} \left( \frac{1}{\omega} \right) - i \pi \delta(\omega) \]
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$$
\begin{bmatrix}
    \mathcal{M} - \frac{i}{2} \Gamma
\end{bmatrix}_{ij} = M M \delta^{(0)}_{ij} + \frac{1}{2 M M} \sum_n \frac{\langle M_i | H_{\text{weak}} | n \rangle \langle n | H_{\text{weak}} | M_j \rangle}{M^{(0)}_M - E_n + i \epsilon} + \ldots
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The Hamilton Matrix: Computing Mixing Parameters

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- Weak interaction sets scale: “Wigner-Weisskopf” approximation
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$$\frac{1}{\omega + i\epsilon} = \mathcal{P} \left( \frac{1}{\omega} \right) - i\pi \delta(\omega)$$
Calculation of $\Gamma_{12}$

Absorptive Part

\[
\Gamma_{ij} = \frac{1}{2M_M} \sum_n \langle M_i | \mathcal{H}_{\text{weak}} | n \rangle \langle n | \mathcal{H}_{\text{weak}} | M_j \rangle (2\pi) \delta(M_M^{(0)} - E_n)
\]

- On-shell production of intermediate particles
- $i = j$ recovers total width
- Dominated by $\Delta B = 1$ operator
- Only $u$ and $c$ intermediate state quarks

Calculation

- Perturbative corrections
  1. Up to NLO in $\alpha_s(m_b)$
  2. All-order summation of $\alpha_s^n(m_c^2/m_b^2)^n \log(m_c^2/m_b^2)$
- Non-perturbative corrections up to $\Lambda_{\text{QCD}}/m_b$ (5 more operators)
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[arXiv:1102.4274]
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\[
M_{ij} = \frac{1}{2M_M} \langle M_i | \mathcal{H}_{\Delta F=2} | M_j \rangle + \frac{1}{2M_M} \sum_n \mathcal{P} \frac{\langle M_i | \mathcal{H}_{\Delta F=1} | n \rangle \langle n | \mathcal{H}_{\Delta F=1} | M_j \rangle}{M_M^{(0)} - E_n}
\]

- dominated by $\Delta B = 2$ operator
- top quark dominate intermediate state

**Result within SM**

\[
M_{12} = \frac{G_F^2 M_B}{12\pi^2} M_W^2 (V_{tb} V_{ts}^*)^2 \hat{\eta}_B S_0 \left( \frac{m_t^2}{M_W^2} \right) f_{B_s}^2 B
\]

- Lattice determination of Bag parameter $B$ and decay constant $f_{B_s}$
- Mass difference measured precisely

⇒ $|M_{12}|$ for $|M_{12}| \gg \Gamma_{12}$
Calculation of $M_{12}$

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$\Rightarrow$ Fixes $|M_{12}|$ for $|M_{12}| \gg \Gamma_{12}$


**Calculation of $A_{SL}$**

**In the Standard Model**

- Naming scheme for $B_{s,d}$: $A_{SL}^{d,s}$
- Sum of both including production asymmetry: $A_{SL}^{b}$
- SM phase originates from CKM mechanism (convention dependent)

\[
A_{SL}^{d,s} \approx \text{Im} \frac{\Gamma_{12}^{d,s}}{M_{12}^{d,s}} \approx \frac{\Delta \Gamma_{d,s}}{\Delta M_{d,s}} \tan \phi_{d,s}
\]

- Highly suppressed: $|\Gamma_{12}/M_{12}| = \mathcal{O}(m_b^2/M_W^2, m_c^2/m_b^2)$

**Beyond the Standard Model**

- Additional phases can be introduced in $M_{12}$ due to New Physics
  
  \(\Rightarrow\) Introduces sensitivity to $\text{Re}(\Gamma_{12}/M_{12})_{SM}$
  
  \(\Rightarrow\) Enhanced sensitivity for BSM physics in this observable
Calculation of $A_{sL}$

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Standard Model Predictions \cite{A.Lenz,U.Nierste: 1102.4274,hep-ph/0612167}

Predictions for $B_s$ System

- $2|\Gamma_{12}^s| = (0.087 \pm 0.021) \text{ ps}^{-1}$
- $\Delta M_s = (17.3 \pm 2.6) \text{ ps}^{-1}$
- $\phi_s = (0.22 \pm 0.06)^\circ$
- $a_{SL}^s = (1.9 \pm 0.3) \times 10^{-5}$

Predictions for $B_d$ System

- $2|\Gamma_{12}^d| = (2.74 \pm 0.51) \times 10^{-3} \text{ ps}^{-1}$
- $\phi_d = (-4.3 \pm 1.4)^\circ$
- $a_{SL}^d = -(4.1 \pm 0.6) \times 10^{-4}$
## Standard Model Predictions

### Predictions for $B_s$ System
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Current Experimental Situation

DØ Like-sign Di-muon measurement

\[ A_{SL}^b = -[7.87 \pm 1.72 \text{ (stat)} \pm 0.93 \text{ (syst)}] \times 10^{-3} \]
\[ = (0.594 \pm 0.022) A_{SL}^d + (0.406 \pm 0.022) A_{SL}^s \]

\[ \Delta M_s = (17.719 \pm 0.043) \text{ ps}^{-1} \text{ [hep-ex/0609040,LHCb-CONF-2011-050(005)]} \]
\[ \Delta M_d = (0.507 \pm 0.004) \text{ ps}^{-1} \text{ Heavy Flavor Averaging Group (HFAG)} \]
\[ \Delta \Gamma_s = (0.116 \pm 0.019) \text{ ps}^{-1} \text{ LHCb} \]
\[ \Delta \Gamma_s = (0.068 \pm 0.027) \text{ ps}^{-1} \text{ CDF} \]
\[ \Delta \Gamma_s = (0.163^{+0.065}_{-0.064}) \text{ ps}^{-1} \text{ DØ} \]

LHCb measurement of time dependent CP asymmetry

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New measurement tend to agree well with SM
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- New measurement tend to agree well with SM
How Can New Physics Enter?
- Can induce new operator structures
- Can modify Wilson coefficients

Parametrization and Effects
- For $B$ mesons: $|\Gamma_{12}/M_{12}| \ll 1$

\[ \Delta m = |2M_{12}|, \quad \Delta \Gamma = 2|\Gamma_{12}| \cos \phi_{12}, \quad A_{\text{sl}} = 2\delta = \text{Im}(\Gamma_{12}/M_{12}) \]

- In general: Changing phase and absolute value possible

\[ M_{12} = M_{12}^{\text{SM}} |\Delta_s| e^{i\phi_s^\Delta} \]

1. Only relative phase relevant
2. $M_{12}$: Heavy intermediate particles, but $|M_{12}|$ constrained by exp.
3. $\Gamma_{12}$ dominated by on-shell charm intermediate states
   \Rightarrow Believed not to change dramatically by New Physics contributions
Physical Constraints

- Mass and width of physical states have to be positive
- Unitarity has to be conserved
- Time evolution of any linear combination of $|B^0\rangle$ and $|\bar{B}^0\rangle$ determined entirely by the $\Gamma$ matrix
  \[ \Rightarrow \Gamma \text{ itself has positive eigenvalues} \]
- Defining $\Gamma = (\Gamma_H + \Gamma_L)/2$, $x = (m_H - m_L)/\Gamma$ and $y = (\Gamma_L - \Gamma_H)/(2\Gamma)$
  \[ \delta^2 < \frac{\Gamma_H \Gamma_L}{(m_H - m_L)^2 + (\Gamma_H + \Gamma_L)^2/4} = \frac{1 - y^2}{1 + x^2} \]
- Known as unitarity bound or Bell-Steinberger inequality
  
Generic Conditions on Mixing Parameter

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$\Rightarrow$ $\Gamma$ itself has positive eigenvalues

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\[J.S. \ Bell, \ J. \ Steinberger, \ “Weak \ interactions \ of \ kaons”, \ in \ R. \ G. \ Moorhouse \ et \ al., \ Eds., \ Proceedings \ of \ the \ Oxford \ Int. \ Conf. \ on \ Elementary \ Particles, \ Rutherford \ Laboratory, \ Chilton, \ England, \ 1965, \ p. \ 195.\]
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- Mass and width of physical states have to be positive
- Unitarity has to be conserved
- Time evolution of any linear combination of $|B^0\rangle$ and $|\bar{B}^0\rangle$ determined entirely by the $\Gamma$ matrix
  \[ \Rightarrow \Gamma \text{ itself has positive eigenvalues} \]
- Defining $\Gamma = (\Gamma_H + \Gamma_L)/2$, $x = (m_H - m_L)/\Gamma$ and $y = (\Gamma_L - \Gamma_H)/(2\Gamma)$
  \[ \delta^2 < \frac{\Gamma_H \Gamma_L}{(m_H - m_L)^2 + (\Gamma_H + \Gamma_L)^2/4} = \frac{1 - y^2}{1 + x^2} \]

- Known as unitarity bound or Bell-Steinberger inequality
Reminder: Sketching Derivation of Unitarity Bound

**Definitions**

1. \( a_i = \sqrt{2\pi \rho_i} \langle f_i | H | B \rangle \), \( \bar{a}_i = \sqrt{2\pi \rho_i} \langle f_i | H | \bar{B} \rangle \)

\[ \Rightarrow a_i^* a_i = \Gamma_{11}, \quad \bar{a}_i^* \bar{a}_i = \Gamma_{22}, \quad \bar{a}_i^* a_i = \Gamma_{12} \]

2. In the physical basis we have

\[ a_i = \frac{1}{2p} (a_{Hi} + a_{Li}), \quad \bar{a}_i = \frac{1}{2q} (a_{Li} - a_{Hi}) \]

\[ \Rightarrow a_{Hi}^* a_{Hi} = \Gamma_H, \quad a_{Li}^* a_{Li} = \Gamma_L, \quad a_{Hi}^* a_{Li} = -i(m_H - m_L + i\Gamma) \delta \]

**Sketch of Derivation**

1. Apply optical theorem and Cauchy Schwartz inequality

\[ \Rightarrow \text{Forces } \Gamma \text{ to have positive semi-definit eigenvalues} \]

\[ \Gamma_{11} \geq |\Gamma_{12}| \]

2. Apply the same to the physical basis with unitarity condition

3. Leads to the unitarity bound \( \delta^2 < \frac{\Gamma_H \Gamma_L}{(m_H - m_L)^2 + (\Gamma_H + \Gamma_L)^2 / 4} = \frac{1 - y^2}{1 + x^2} \)
Definition:

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### Implication for the Three Neutral Mesons

#### Neutral $K$ Mesons
- Very limited amount of final states
- Unitarity bound developed for this case

#### Neutral $D$ Mesons
- Difficult in theory as well as in experiment
  1. Non-perturbative methods?
  2. Huge GIM suppression
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- Lots of possible final states
- Experimental precision is increasing
- Systematic expansion for theoretical calculations possible
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- Assume knowledge of $|\Gamma_{12}|$
  - Can be computed in a reliable, systematical expansion in the $B$ system
- Define $y_{12} \geq 0$ with
  
  \[
  0 \leq y_{12} = \frac{|\Gamma_{12}|}{\Gamma} \leq 1
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Goals

- Use precise measurement of mass difference
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Sketch of the Steps

- Start with the same equations as for the unitarity bound
- Use $y_{12}$ instead of unitarity constraint
- Proceed with same steps

The Result

$$\delta^2 = \frac{y_{12}^2 - y^2}{y_{12}^2 + x^2} = \frac{|\Gamma_{12}|^2 - (\Delta \Gamma)^2/4}{|\Gamma_{12}|^2 + (\Delta m)^2}$$

- Entirely determined by solving the eigenstate problem
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Obtain a Physical Understanding

- Relation can also be obtained from a scaling argument
  - $\delta$ depends only on mixing parameters and independent of $\Gamma$
  - Scale $\Gamma$ by $y_{12}$
    1. Does not affect $\delta$
    2. Changes $x \rightarrow x/y_{12}$ and $y \rightarrow y/y_{12}$

$\Rightarrow$ Combining argument with unitarity bound recovers exact relation

Derivation not Assuming CPT invariance

- Assuming no CPT invariance implies $M_{11} \neq M_{22}$ and $\Gamma_{11} \neq \Gamma_{22}$
- Mixing parameters depend on difference of diagonal components

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Combined Bound on $A_{\text{SL}}^b$

**Experimental Situation**
- Hadron colliders produce admixture of $B_s$ and $B_d$
- Production asymmetry is known at DØ
  \[ A_{\text{SL}}^b = (0.594 \pm 0.022) A_{\text{SL}}^d + (0.406 \pm 0.022) A_{\text{SL}}^s \]
- $B$ factories can access $B_d$ ⇒ need Super-B for sufficient precision

**Implication for Unitarity Relation with Theory Input**
- Relation (bound) on $|\delta|$ 
- Relation for individual $|A_{\text{SL}}^{d,s}|$
- With know production asymmetry we can give a bound on 
  \[ |A_{\text{SL}}^b| \leq (1.188 \pm 0.044) \delta_{\text{max}}^d + (0.812 \pm 0.044) \delta_{\text{max}}^s \]
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Plot of the Bound

- Assuming $\Delta \Gamma_d = 0$
- Horizontal lines: $1\sigma$ range of $|A_{SL}^b|$
  - DØ measurement
- Vertical lines correspond to $\Delta \Gamma_s$
  - LHCb measurement

- Shaded regions are allowed by theory prediction
  - Darker uses $1\sigma$ upper range
  - Lighter uses $2\sigma$ upper range
- Dashed [dotted] curves: Mixed sigma interval of theory predictions
- The vertical boundaries of the shaded regions arise because $|\Delta \Gamma_s| > 2|\Gamma_{12}^s|$ is unphysical.
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Plot on Individual Bound on $B_s$

**Interpretation**
- Horizontal lines correspond to LHCb measurement
- Dark [light] shaded allowed by $1\sigma$ [$2\sigma$] theory variation
- No discrepancy claimed in experiment
Plot on Individual Bound on $B_d$

Interpretation

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Numerical Interpretation

Remarks

- Problematic: $|\Delta \Gamma_s|^{\text{meas.}} > 2 |\Gamma_{12}^s|$ is unphysical
- Numerator of Relation can vanish $\Rightarrow$ Upper bound
- Assume 2$\sigma$ theory prediction for a conservative estimate

Results

- For $B_s$ system, we obtain by propagating the uncertainties, taking into account the unphysical region
  $|A_{SL}^s| < 4.2 \times 10^{-3}$
- 2-3 times better than best current experimental bound
- For the $B_d$ system we obtain a comparable bound
  $|A_{SL}^d| < 7.4 \times 10^{-3}$
- Significant improvement possible by observing $|\Delta \Gamma_d| > 0$
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Strength of the Bound

- Upper bound on $y_{12}$ implies an upper bound on $|\delta|$.
- Relation is much stronger for small $y_{12}$, as e.g. in the $B_d$ system.

Comparing to Known Results

- DØ $A_{\text{SL}}$ measurement: $3.9\sigma$ discrepancy with SM
  - Correlated with the discrepancy found in our analysis
    1. SM prediction of $A_{\text{SL}}$ uses calculation of $|\Gamma_{12}|$
    2. The relation uses $|\Gamma_{12}|$ as an input
    3. Calculation of $|\Gamma_{12}|$ and $\text{Im}(\Gamma_{12})$ rely on the same OPE
- Large cancellations in $\text{Im}(\Gamma_{12})$ ⇒ Uncertainties could be larger than expected from NLO calculation [hep-ph/0308029, hep-ph/0307344]
- The sensitivity of $\Gamma_{12}$ to NP is generally weak
- Interesting to determine $\delta$ additionally from this relation.
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No Go Theorem (preliminary)

The Claim

There is no generic bound, stronger than the unitarity bound

Sketch of Derivation

- Unitarity bound is saturated if
  \[ \langle f|T|B_H \rangle \propto \langle f|T|B_L \rangle \]
- Start with an arbitrary, generic decaying two-state system
- Wigner-Weisskopf approximation: Any choice of parameters OK
  \[ \Rightarrow \text{Orthogonal, non CP violating system as starting point} \]
- Arbitrary new UV physics can change \( M_{12} \) independently of \( \Gamma_{12} \)
- Varying \( M_{12} \) keeping mass and width of states physical
  \[ \Rightarrow \text{Unitarity bound can be saturated (relax constraint Arg } M_{12} = \text{Arg } \Gamma_{12} \) \]
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  - Arbitrary new UV physics can change \( M_{12} \) independently of \( \Gamma_{12} \)
  - Varying \( M_{12} \) keeping mass and width of states physical
  \[ \Rightarrow \text{Unitarity bound can be saturated (relax constraint Arg } M_{12} = \text{Arg } \Gamma_{12}) \]
  - Explicit mathematical check
Summary

- Provided a physical derivation of the exact relation allowing for theoretical input on $|\Gamma_{12}|$
  1. Input is typically insensitive to New Physics
  2. Avoids largest uncertainties of theory calculation
  3. Valid even if CPT is violated

- Independent of the discrepancy found from a global fit
  1. Application to $B_{d,s}$ systems leads to the individual bounds
    
    $$|A_{SL}^s| < 4.2 \times 10^{-3} \quad |A_{SL}^d| < 7.4 \times 10^{-3}$$

  2. Providing a bound on the individual asymmetries at comparable or better levels than the current experimental bounds

  3. Bounds are in tension with the DØ measurement of $A_{SL}^b$

- Once an unambiguous determination of $A_{SL}$ or $\Delta \Gamma$ is made, we can use it to constrain the other observable.
Backup Slides
Direct CP Violation

General Comment

- Need CP even and odd phases
  \[ \Gamma \propto |A_1(f) + A_2(f)|^2 \]
- Interference of CP conserving (strong) and violating (weak) phases
- Occurs in neutral and charged meson decays
- Necessary condition: \( |A(f)| \neq |\bar{A}(\bar{f})| \)

Example of Process

Only source of CP violation in charged meson decays

\[ A_{f^\pm} \equiv \frac{\Gamma(P^- \to f^-) - \Gamma(P^+ \to f^+)}{\Gamma(P^- \to f^-) + \Gamma(P^+ \to f^+)} = \frac{|\bar{A}(f^-)/A(f^+)|^2 - 1}{|\bar{A}(f^-)/A(f^+)|^2 + 1} \]
CP Violation in Interference of Mixing and Decay

General Comments [Bigi, Sanda: CP violation]

- Interference between decay and mixing to common final state
- Necessary condition:

$$\text{Im} \left[ \frac{q}{p} \frac{\ddot{A}(f)}{A(f)} \right] \neq 0$$

Example of Process

- CP Asymmetry (easy form only in limits, e.g. B mesons)

$$A_{f_{CP}}(t) \equiv \frac{\Gamma(\bar{P}_0 \rightarrow f_{CP}) - \Gamma(P_0 \rightarrow f_{CP})}{\Gamma(\bar{P}_0 \rightarrow f_{CP}) + \Gamma(P_0 \rightarrow f_{CP})}$$

$$= -A_{CP}^{\text{dir}} \cos(\Delta M t) - A_{CP}^{\text{mix}} \sin(\Delta M t)$$

$$= \frac{\text{cosh}(\Delta \Gamma t/2) + A_{\Delta \Gamma} \sinh(\Delta \Gamma t/2)}{\text{cosh}(\Delta \Gamma t/2) + A_{\Delta \Gamma} \sinh(\Delta \Gamma t/2)}$$

- $B_s \rightarrow J/\Psi \phi$ and $B_s \rightarrow J/\Psi f_0$