The Frobenius group $T_7$ as a symmetry in the lepton sector

Ulrike Regner, University of Vienna

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The Frobenius group $T_7$ as a symmetry in the lepton sector

Introduction

Overview

- Introduction
- The discrete symmetry group $T_7$
- Yukawa terms and mass matrices
- The neutrino mixing matrix
- Interactions
- The scalar potential of $\Phi$
- Conclusions
Introduction

Neutrinos are puzzling:

- experimentally found properties (i.e. nonzero mass differences) cannot be included in the SM without righthanded neutrinos
- mixing angles (one vanishingly small, others large) cannot be explained
- absolute masses still unknown
- smallness of masses cannot be explained
- mass spectrum cannot be explained
- Dirac or Majorana nature is unsolved
- extremely hard to measure due to small interaction rates
- ...

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Introduction

Favoured:
Tribimaximal neutrino mixing matrix (good agreement with experimental data)

\[ U_{PMNS} = \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\end{pmatrix} \]  

⇒ underlying family symmetry: discrete subgroups of $SU(3)$

Smallest group with 3-dim irreps: $A_4$
Smallest group with two nonequivalent 3-dim irreps: $T_7$, suggested by Cao, Khalil, Ma, Okada (2010)
The Frobenius group \( T_7 \) as a symmetry in the lepton sector

The discrete symmetry group \( T_7 \)

Properties of \( T_7 \)

Elements of \( T_7 \)

21 elements, two generators:

\[
a = \begin{pmatrix}
\rho & 0 & 0 \\
0 & \rho^2 & 0 \\
0 & 0 & \rho^4
\end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0
\end{pmatrix}
\]

\[
\rho = e^{\frac{2i\pi}{7}}
\]

Important identities:

\[
a^7 = e, \ b^3 = e, \ ba = a^2b
\]
The Frobenius group $T_7$ as a symmetry in the lepton sector

The discrete symmetry group $T_7$

Properties of $T_7$

Elements of $T_7$

$e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ \hspace{1cm} $b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$ \hspace{1cm} $b^2 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$a = \begin{pmatrix} \rho & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \end{pmatrix}$ \hspace{1cm} $ab = \begin{pmatrix} 0 & \rho & 0 \\ 0 & 0 & \rho^2 \\ \rho^4 & 0 & 0 \end{pmatrix}$ \hspace{1cm} $ab^2 = \begin{pmatrix} 0 & 0 & \rho \\ \rho^2 & 0 & 0 \\ 0 & \rho^4 & 0 \end{pmatrix}$

$a^2 = \begin{pmatrix} \rho^2 & 0 & 0 \\ 0 & \rho^4 & 0 \\ 0 & 0 & \rho \end{pmatrix}$ \hspace{1cm} $a^2b = \begin{pmatrix} 0 & \rho^2 & 0 \\ 0 & 0 & \rho^4 \\ \rho & 0 & 0 \end{pmatrix}$ \hspace{1cm} $a^2b^2 = \begin{pmatrix} 0 & 0 & \rho^2 \\ \rho^4 & 0 & 0 \\ 0 & \rho & 0 \end{pmatrix}$

$a^3 = \begin{pmatrix} \rho^3 & 0 & 0 \\ 0 & \rho^6 & 0 \\ 0 & 0 & \rho^5 \end{pmatrix}$ \hspace{1cm} $a^3b = \begin{pmatrix} 0 & \rho^3 & 0 \\ 0 & 0 & \rho^6 \\ \rho^5 & 0 & 0 \end{pmatrix}$ \hspace{1cm} $a^3b^2 = \begin{pmatrix} 0 & 0 & \rho^3 \\ \rho^6 & 0 & 0 \\ 0 & \rho^5 & 0 \end{pmatrix}$

$a^4 = \begin{pmatrix} \rho^4 & 0 & 0 \\ 0 & \rho^2 & 0 \\ 0 & 0 & \rho \end{pmatrix}$ \hspace{1cm} $a^4b = \begin{pmatrix} 0 & \rho^4 & 0 \\ 0 & 0 & \rho \\ \rho^2 & 0 & 0 \end{pmatrix}$ \hspace{1cm} $a^4b^2 = \begin{pmatrix} 0 & 0 & \rho^4 \\ \rho & 0 & 0 \\ 0 & \rho^2 & 0 \end{pmatrix}$

$a^5 = \begin{pmatrix} \rho^5 & 0 & 0 \\ 0 & \rho^3 & 0 \\ 0 & 0 & \rho^6 \end{pmatrix}$ \hspace{1cm} $a^5b = \begin{pmatrix} 0 & \rho^5 & 0 \\ 0 & 0 & \rho^3 \\ \rho^6 & 0 & 0 \end{pmatrix}$ \hspace{1cm} $a^5b^2 = \begin{pmatrix} 0 & 0 & \rho^5 \\ \rho^3 & 0 & 0 \\ 0 & \rho^6 & 0 \end{pmatrix}$

$a^6 = \begin{pmatrix} \rho^6 & 0 & 0 \\ 0 & \rho^5 & 0 \\ 0 & 0 & \rho^3 \end{pmatrix}$ \hspace{1cm} $a^6b = \begin{pmatrix} 0 & \rho^6 & 0 \\ 0 & 0 & \rho^5 \\ \rho^3 & 0 & 0 \end{pmatrix}$ \hspace{1cm} $a^6b^2 = \begin{pmatrix} 0 & 0 & \rho^6 \\ \rho^5 & 0 & 0 \\ 0 & \rho^3 & 0 \end{pmatrix}$
The Frobenius group $T_7$ as a symmetry in the lepton sector

The discrete symmetry group $T_7$

Properties of $T_7$

Irreducible representations of $T_7$

$$\sum_i d_i^2 = |G|$$

<table>
<thead>
<tr>
<th>dim.</th>
<th>name</th>
<th>$a$</th>
<th>$b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1_1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1_2</td>
<td>1</td>
<td>$\omega$</td>
</tr>
<tr>
<td>1</td>
<td>1_3</td>
<td>1</td>
<td>$\omega^2$</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>$\begin{pmatrix} \rho &amp; 0 &amp; 0 \ 0 &amp; \rho^2 &amp; 0 \ 0 &amp; 0 &amp; \rho^4 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \ 1 &amp; 0 &amp; 0 \end{pmatrix}$</td>
</tr>
<tr>
<td>3</td>
<td>3*</td>
<td>$\begin{pmatrix} \rho^6 &amp; 0 &amp; 0 \ 0 &amp; \rho^5 &amp; 0 \ 0 &amp; 0 &amp; \rho^3 \end{pmatrix}$</td>
<td>$\begin{pmatrix} 0 &amp; 1 &amp; 0 \ 0 &amp; 0 &amp; 1 \ 1 &amp; 0 &amp; 0 \end{pmatrix}$</td>
</tr>
</tbody>
</table>

$$\omega = e^{\frac{2i\pi}{3}}$$
The Frobenius group $T_7$ as a symmetry in the lepton sector

Properties of $T_7$

Tensor products of the irreps of $T_7$

<table>
<thead>
<tr>
<th>$1_p \otimes 1_q$</th>
<th>$\cong 1_{(p+q) \mod(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1_p \otimes 3$</td>
<td>$\cong 3$</td>
</tr>
<tr>
<td>$1_p \otimes 3^*$</td>
<td>$\cong 3^*$</td>
</tr>
<tr>
<td>$3 \otimes 3$</td>
<td>$\cong 3 \oplus 3^* \oplus 3^*$</td>
</tr>
<tr>
<td>$3^* \otimes 3^*$</td>
<td>$\cong 3^* \oplus 3 \oplus 3$</td>
</tr>
<tr>
<td>$3 \otimes 3^*$</td>
<td>$\cong 3^* \oplus 3 \oplus 1_1 \oplus 1_2 \oplus 1_3$</td>
</tr>
</tbody>
</table>
The Frobenius group $T_7$ as a symmetry in the lepton sector

The discrete symmetry group $T_7$

Properties of $T_7$

Conjugacy classes of $T_7$

<table>
<thead>
<tr>
<th>name</th>
<th># elements</th>
<th>elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_1$</td>
<td>1</td>
<td>e</td>
</tr>
<tr>
<td>$C_2$</td>
<td>3</td>
<td>$a, a^2, a^4$</td>
</tr>
<tr>
<td>$C_3$</td>
<td>3</td>
<td>$a^3, a^5, a^6$</td>
</tr>
<tr>
<td>$C_4$</td>
<td>7</td>
<td>$b, ab, a^2b, a^3b, a^4b, a^5b, a^6b$</td>
</tr>
<tr>
<td>$C_5$</td>
<td>7</td>
<td>$b^2, ab^2, a^2b^2, a^3b^2, a^4b^2, a^5b^2, a^6b^2$</td>
</tr>
</tbody>
</table>
The Frobenius group $T_7$ as a symmetry in the lepton sector

The discrete symmetry group $T_7$

Properties of $T_7$

Character table of $T_7$

<table>
<thead>
<tr>
<th></th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>ord($g$)</td>
<td>1</td>
<td>7</td>
<td>7</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$1_1$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$1_2$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$\omega$</td>
<td>$\omega^2$</td>
</tr>
<tr>
<td>$1_3$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>$\omega^2$</td>
<td>$\omega$</td>
</tr>
<tr>
<td>$3$</td>
<td>3</td>
<td>$\eta$</td>
<td>$\eta^*$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$3^*$</td>
<td>3</td>
<td>$\eta^*$</td>
<td>$\eta$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[
\omega = e^{\frac{2i\pi}{3}}
\]

\[
\eta = -\frac{1}{2} + i\frac{\sqrt{7}}{2}
\]
The Frobenius group $T_7$ as a symmetry in the lepton sector

The discrete symmetry group $T_7$ as a Frobenius group

Frobenius groups: Definitions

Definition

Let $G$ be a finite group that contains a nontrivial subgroup $H$ with the property that $H \cap H^g = \{e\}$, where $e$ is the neutral element of $G$ and $\forall g \in G \setminus H$, $H^g$ is defined as $H^g := \{g^{-1}hg | h \in H\}$. Then $G$ is a **Frobenius group** and the subgroup $H$ is called a **Frobenius complement** of $G$.

Definition

$K = \left( G \setminus \bigcup_{g \in G} H^g \right) \cup \{e\}$ is called the **Frobenius kernel** of $G$ with respect to $H$. 

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The Frobenius group $T_7$ as a symmetry in the lepton sector

The discrete symmetry group $T_7$

$T_7$ as a Frobenius group

Frobenius groups: Theorems

**Theorem**

*Theorem of Frobenius*

$K \leq G$.

**Theorem**

$G \cong K \rtimes H$ as a semidirect product.
The Frobenius group $T_7$ as a symmetry in the lepton sector

The discrete symmetry group $T_7$

$T_7$ as a Frobenius group

$T_7$ as a Frobenius group

$H = \{e, b, b^2\}$ is a Frobenius complement of $T_7$.

The Frobenius kernel of $T_7$ with respect to $H$ is

$$K = (T_7 \setminus \bigcup_{g \in T_7} H^g) \cup \{e\} = \{e, a, a^2, a^3, a^4, a^5, a^6\}. \quad (5)$$

Every element $g \in T_7$ can be written as $kh$, $T_7 \cong K \rtimes H$. 

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The Frobenius group $T_7$ as a symmetry in the lepton sector

Yukawa terms and mass matrices

Transformation properties of the particles

Model suggested by Cao, Khalil, Ma, Okada (2010)

\[
\text{field} & | \quad L_L,i \quad | \quad \bar{\ell}_{R,i} \quad | \quad \nu_{R,i} \quad | \quad \Phi_i \quad | \quad \tilde{\Phi}_i \quad | \quad \chi_i \quad | \quad \eta_i \\
SU(2)_L & | \quad 2 \quad | \quad 1 \quad | \quad 1 \quad | \quad 2 \quad | \quad 2 \quad | \quad 1 \quad | \quad 1 \\
Y & | \quad -1 \quad | \quad +2 \quad | \quad 0 \quad | \quad +1 \quad | \quad -1 \quad | \quad 0 \quad | \quad 0 \\
T_7 & | \quad 3 \quad | \quad 1_i \quad | \quad 3 \quad | \quad 3 \quad | \quad 3^* \quad | \quad 3 \quad | \quad 3^* \\
Y_{B-L} & | \quad -1 \quad | \quad 1 \quad | \quad -1 \quad | \quad 0 \quad | \quad 0 \quad | \quad -2 \quad | \quad -2 \\
\]

\[
\Phi_i = \begin{pmatrix}
\Phi_i^+ \\
\Phi_i^0 \\
\Phi_i^-
\end{pmatrix}
\]
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Yukawa terms and mass matrices

Goal: Mass matrices

Invariant Yukawa terms

Charged leptons

- $\phi$
  - charged lepton mass matrix $M_l$

Dirac mass matrix $M_D$

Neutrinos

- $\eta$
  - $M_\eta$
- $\chi$
  - $M_x$

$M_h$
The Frobenius group $T_7$ as a symmetry in the lepton sector

Yukawa terms and mass matrices

Charged lepton mass matrix

Yukawa terms for the charged leptons

Yukawa terms $\bar{L}_L \Phi \ell_R$ and $\bar{\ell}_R \Phi^\dagger L_L$

$$\Rightarrow$$ mass terms $\bar{\ell}_L M_\ell \ell_R$ and $\bar{\ell}_R M^\dagger_\ell \ell_L$

Tensor products:

$$f_1(\bar{L}_{L,1} \Phi_1 + \bar{L}_{L,2} \Phi_2 + \bar{L}_{L,3} \Phi_3)\ell_{R,1} +$$
$$f_2(\bar{L}_{L,1} \Phi_1 + \omega^2 \bar{L}_{L,2} \Phi_2 + \omega \bar{L}_{L,3} \Phi_3)\ell_{R,2} +$$
$$f_3(\bar{L}_{L,1} \Phi_1 + \omega \bar{L}_{L,2} \Phi_2 + \omega^2 \bar{L}_{L,3} \Phi_3)\ell_{R,3}$$

(6)
The Frobenius group $T_7$ as a symmetry in the lepton sector

Yukawa terms and mass matrices

Charged lepton mass matrix

Invariance under $T_7$

Sums inside brackets transform as 1-dimensional representations:

\begin{align}
(\bar{L}_{L,1}\phi_1 + \bar{L}_{L,2}\phi_2 + \bar{L}_{L,3}\phi_3) &\sim 1_1, \\
(\bar{L}_{L,1}\phi_1 + \omega^2\bar{L}_{L,2}\phi_2 + \omega \bar{L}_{L,3}\phi_3) &\sim 1_2, \\
(\bar{L}_{L,1}\phi_1 + \omega \bar{L}_{L,2}\phi_2 + \omega^2 \bar{L}_{L,3}\phi_3) &\sim 1_3.
\end{align}

$\ell_{R,i}$ outside brackets: $\ell_{R,1} \sim 1_1$, $\ell_{R,2} \sim 1_3$, $\ell_{R,3} \sim 1_2$

$\Rightarrow$ invariance under the group $T_7$
The Frobenius group $T_7$ as a symmetry in the lepton sector

Yukawa terms and mass matrices

Charged lepton mass matrix

**Mass matrix for the charged leptons**

Read off charged lepton mass matrix:

$$M_\ell = \begin{pmatrix} f_1 v_1 & f_2 v_1 & f_3 v_1 \\ f_1 v_2 & f_2 \omega^2 v_2 & f_3 \omega v_2 \\ f_1 v_3 & f_2 \omega v_3 & f_3 \omega^2 v_3 \end{pmatrix} \quad (8)$$

$v_i$ are VEV of $\Phi_i^0$

Hermitian conjugate:

$$M_\ell^\dagger = \begin{pmatrix} f_1^* v_1^* & f_1^* v_2^* & f_1^* v_3^* \\ f_2^* v_1^* & f_2^* \omega v_2^* & f_2^* \omega^2 v_3^* \\ f_3^* v_1^* & f_3^* \omega^2 v_2^* & f_3^* \omega v_3^* \end{pmatrix} \quad (9)$$
The Frobenius group $T_7$ as a symmetry in the lepton sector

Yukawa terms and mass matrices

Neutrino mass matrix

Yukawa terms for the neutrinos

Goal: Find seesaw mass matrix

$$M_{\text{light}} = -M_D^T (M_h^*)^{-1} M_D$$

(10)

Yukawa terms $-\bar{\nu}_R \tilde{\Phi}^\dagger L_L$ and $-\bar{L}_L \tilde{\Phi} \nu_R$

$\Rightarrow$ Dirac mass terms $\bar{\nu}_R M_D \nu_L$ and $\bar{\nu}_L M_D^\dagger \nu_R$

Symmetry $\rightarrow$ all coupling constants need to be equal,

$f_{D,1} = f_{D,2} = f_{D,3} =: f_D$
The Frobenius group $T_7$ as a symmetry in the lepton sector

Yukawa terms and mass matrices

Neutrino mass matrix

Dirac mass matrix for the neutrinos

Dirac neutrino mass matrix:

$$M_D = f_D^* \begin{pmatrix} 0 & 0 & \nu_3 \\ \nu_1 & 0 & 0 \\ 0 & \nu_2 & 0 \end{pmatrix}$$  \hspace{1cm} (11)$$

Hermitian conjugate:

$$M_D^{\dagger} = f_D \begin{pmatrix} 0 & \nu_1^* & 0 \\ 0 & 0 & \nu_2^* \\ \nu_3^* & 0 & 0 \end{pmatrix}$$  \hspace{1cm} (12)$$
Additional Yukawa terms for the neutrinos

Two more invariant Yukawa terms:

\[ \chi^\dagger \nu_R^T C^{-1} \nu_R \quad \text{and} \quad \eta^\dagger \nu_R^T C^{-1} \nu_R \]  \hspace{1cm} (13)

and their hermitian conjugates

\[ \nu_R^\dagger C \nu_R^* \chi \quad \text{and} \quad \nu_R^\dagger C \nu_R^* \eta \]  \hspace{1cm} (14)

\[ \Rightarrow \text{ heavy Majorana mass matrix } M_h = M_\chi + M_\eta \]

\[ B - L \text{ quantum number crucial here: guarantees that } \chi \text{ and } \eta \text{ are independent fields} \]
The Frobenius group $T_7$ as a symmetry in the lepton sector

Yukawa terms and mass matrices

Neutrino mass matrix

Majorana mass matrix for the neutrinos, part 1

\[ M_{\chi} = h \begin{pmatrix} u_2^* & 0 & 0 \\ 0 & u_3^* & 0 \\ 0 & 0 & u_1^* \end{pmatrix} \] (15)

$u_i$...VEV of $\chi_i$, $h$...coupling constant of $\nu_{R,i}$ (all equal!)

Hermitian conjugate:

\[ M_{\chi}^\dagger = h^* \begin{pmatrix} u_2 & 0 & 0 \\ 0 & u_3 & 0 \\ 0 & 0 & u_1 \end{pmatrix} \] (16)
The Frobenius group $T_7$ as a symmetry in the lepton sector

Yukawa terms and mass matrices

Neutrino mass matrix

Majorana mass matrix for the neutrinos, part 2

$$M_\eta = h' \begin{pmatrix} 0 & u'_3 & u'_2 \\ u'^*_3 & 0 & u'^*_1 \\ u'^*_2 & u'^*_1 & 0 \end{pmatrix}$$  \hspace{1cm} (17)$$

Hermitian conjugate:

$$M^\dagger_\eta = h'^* \begin{pmatrix} 0 & u'_3 & u'_2 \\ u'_3 & 0 & u'_1 \\ u'_2 & u'_1 & 0 \end{pmatrix}$$  \hspace{1cm} (18)$$

$u'_i$...VEV of $\eta_i$

Assumption: $u_i \sim u'_i$.

Note: $M_\chi$ and $M_\eta$ are symmetric
Yukawa terms and mass matrices

Neutrino mass matrix

Heavy Majorana mass matrix for the neutrinos

\[ M_h = M_\chi + M_\eta = h \begin{pmatrix} u_2^* & 0 & 0 \\ 0 & u_3^* & 0 \\ 0 & 0 & u_1^* \end{pmatrix} + h' \begin{pmatrix} 0 & u_3'^* & u_2'^* \\ u_3'^* & 0 & u_1'^* \\ u_2'^* & u_1'^* & 0 \end{pmatrix} \] (19)

Hermitian conjugate:

\[ M_h^\dagger = M_\chi^\dagger + M_\eta^\dagger = h^* \begin{pmatrix} u_2 & 0 & 0 \\ 0 & u_3 & 0 \\ 0 & 0 & u_1 \end{pmatrix} + h'^* \begin{pmatrix} 0 & u_3' & u_2' \\ u_3' & 0 & u_1' \\ u_2' & u_1' & 0 \end{pmatrix} \] (20)
The Frobenius group $T_7$ as a symmetry in the lepton sector

Yukawa terms and mass matrices

The seesaw mass matrix

Assumptions on the seesaw mass matrix

\[ M_{\text{light}} = -M_D^T (M_h^*)^{-1} M_D \]  \hspace{1cm} (21)

- $u := u_1 = u_2 = u_3 \neq 0 \rightarrow \chi$ breaks in the $(1,1,1)$ direction
- $u'_1 = u'_2 = 0$ but $u'_3 \neq 0 \rightarrow \eta$ breaks in the $(0,0,1)$ direction
- $v_1 = v_2 = v_3 =: v$

$\Rightarrow Z_3 - Z_2$ misalignment
The Frobenius group $T_7$ as a symmetry in the lepton sector

Yukawa terms and mass matrices

The seesaw mass matrix

Simplifying the seesaw mass matrix

Define $A := h^* u$ and $B := h'^* u'_3$

$$M_h = \begin{pmatrix} h^* u & h'^* u'_3 & 0 \\ h'^* u'_3 & h^* u & 0 \\ 0 & 0 & h^* u \end{pmatrix} = \begin{pmatrix} A & B & 0 \\ B & A & 0 \\ 0 & 0 & A \end{pmatrix}$$

Inverse:

$$M_h^{-1} = \frac{1}{A^3 - AB^2} \begin{pmatrix} A^2 & -AB & 0 \\ -AB & A^2 & 0 \\ 0 & 0 & A^2 - B^2 \end{pmatrix}$$
The Frobenius group $T_7$ as a symmetry in the lepton sector

Yukawa terms and mass matrices

The seesaw mass matrix

**Light neutrino mass matrix**

$$M_{\text{light}} = -M_D^T M_h^{-1} M_D = -\frac{(f_D^*)^2 v^2}{A^3 - AB^2} \begin{pmatrix} A^2 & 0 & -AB \\ 0 & A^2 - B^2 & 0 \\ -AB & 0 & A^2 \end{pmatrix}$$  \hspace{1cm} (24)

Assumption on scales already made in the context of $B - L$ gauging:

$$M_D \sim v \text{ and } M_h \sim u \sim u'_3$$

$$v \ll u \rightarrow M_D \ll M_h$$
The Frobenius group $T_7$ as a symmetry in the lepton sector

Yukawa terms and mass matrices

The seesaw mass matrix

The seesaw mass matrix in terms of three independent parameters

Define $C := f_D^* v^2$, $p_1^2 := \frac{CA^2}{A^3 - AB^2}$ and $p_2^2 := \frac{CB^2}{A^3 - AB^2}$, absorb overall phase: $p_1 = e^{i\alpha_1}|p_1|$

$$M_{\text{light}} = e^{2i\alpha_1} \begin{pmatrix} -|p_1|^2 & 0 & e^{-i\alpha_1}|p_1|p_2 \\ 0 & -|p_1|^2 + e^{-2i\alpha_1}p_2^2 & 0 \\ e^{-i\alpha_1}|p_1|p_2 & 0 & -|p_1|^2 \end{pmatrix}$$

(25)

Three real parameters $|p_1|$, $\Im(e^{-i\alpha_1}p_2)$ and $\Re(e^{-i\alpha_1}p_2)$
The Frobenius group $T_7$ as a symmetry in the lepton sector

The neutrino mixing matrix

**PMNS neutrino mixing matrix**

Pontecorvo-Maki-Nakagawa-Sakata (PMNS) neutrino mixing matrix: $V = U_L^\dagger U_\nu$

Diagonalize mass matrices:

$$U_L^\dagger M_\ell U_R^\ell = \hat{M}_\ell$$ (26)

and

$$U_\nu^T M_{\text{light}} U_\nu = \hat{M}_{\text{light}}$$ (27)

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Bidiagonalization of the charged lepton mass matrix

\( U_\ell^L \): bidiagonalize the charged lepton mass matrix:

\[
M_\ell = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \sqrt{3} v \begin{pmatrix} f_1 & 0 & 0 \\ 0 & f_2 & 0 \\ 0 & 0 & f_3 \end{pmatrix}
\] (28)

so

\[
U_\ell^L = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}
\] (29)

and

\[
U_\ell^L^\dagger = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}
\] (30)
Diagonalization of the neutrino mass matrix

Neutrino sector:

$$\tilde{U}_\nu = \begin{pmatrix} r & 0 & -r \\ 0 & 1 & 0 \\ r & 0 & r \end{pmatrix}$$

(31)

where $r := \frac{1}{\sqrt{2}}$

Diagonalizes $M_{\text{light}}$:

$$\hat{M}_{\text{light}} = \tilde{U}_\nu^T M_{\text{light}} \tilde{U}_\nu =$$

$$- \frac{(f_D^*)^2 v^2}{A^3 - AB^2} \begin{pmatrix} A^2 - AB & 0 & 0 \\ 0 & A^2 - B^2 & 0 \\ 0 & 0 & A^2 + AB \end{pmatrix}$$

(32)

$$U_\nu = \tilde{U}_\nu e^{i \beta}$$

(33)
The Frobenius group $T_7$ as a symmetry in the lepton sector

The neutrino mixing matrix

The tribimaximal neutrino mixing matrix

\[ V = U_L^{\ell} U \nu = \]

\[
\frac{1}{\sqrt{3}} \begin{pmatrix}
\frac{\sqrt{2}}{1+\omega^2} & 1 & 0 \\
\frac{1+\omega}{\sqrt{2}} & \omega & \frac{-1+\omega^2}{\sqrt{2}} \\
\frac{1+\omega}{\sqrt{2}} & \omega^2 & \frac{-1+\omega^2}{\sqrt{2}} \\
\end{pmatrix}
\begin{pmatrix}
e^{i\beta_1} & 0 & 0 \\
0 & e^{i\beta_2} & 0 \\
0 & 0 & e^{i\beta_3} \\
\end{pmatrix} = \]

(34)

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & \omega & 0 \\
0 & 0 & -\omega^2 \\
\end{pmatrix}
\begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\end{pmatrix}
\begin{pmatrix}
e^{i\beta_1} & 0 & 0 \\
0 & e^{i\beta_2} & 0 \\
0 & 0 & e^{i\beta_3} \\
\end{pmatrix} \]

(36)
The Frobenius group $T_7$ as a symmetry in the lepton sector

The neutrino mixing matrix

The tribimaximal neutrino mixing matrix

Phases on both sides irrelevant for mixing $\Rightarrow$ tribimaximal mixing!
(Harrison, Perkins, Scott (1999,2002))

Define

$$\tilde{V} = \begin{pmatrix}
\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix},$$

(37)

the mixing matrix without the phases.
The Frobenius group $T_7$ as a symmetry in the lepton sector

The neutrino mixing matrix

The tribimaximal neutrino mixing matrix

General form:

$$
\tilde{V} = \begin{pmatrix}
    c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
    -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\
    s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
$$

(38)

$s_{ij} = \sin(\theta_{ij})$, $c_{ij} = \cos(\theta_{ij})$

Only parameters: the three mixing angles $\theta_{ij}$ and one phase $\delta$
The Frobenius group $T_7$ as a symmetry in the lepton sector

The neutrino mixing matrix

Summary of best fits from experimental data

<table>
<thead>
<tr>
<th>parameter $\sin^2(\theta)$</th>
<th>$\pm 1\sigma$</th>
<th>$\pm 2\sigma$</th>
<th>$\pm 3\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sin^2(\theta_{12})$</td>
<td>0.296 – 0.329</td>
<td>0.280 – 0.347</td>
<td>0.265 – 0.364</td>
</tr>
<tr>
<td>$\sin^2(\theta_{23})$</td>
<td>0.39 – 0.50</td>
<td>0.36 – 0.60</td>
<td>0.34 – 0.64</td>
</tr>
<tr>
<td>$\sin^2(\theta_{13})$</td>
<td>0.018 – 0.032</td>
<td>0.012 – 0.041</td>
<td>0.005 – 0.050</td>
</tr>
</tbody>
</table>

Data from Fogli, Lisi, Marrone, Palazzo, Rotunno (2011)

Tribimaximal still possible, but not realized exactly (radiative corrections)
The Frobenius group $T_7$ as a symmetry in the lepton sector

Interactions

New decays of $\tau^\pm$ implied by the $Z_3$ symmetry

$$\tau^- \rightarrow \mu^- \mu^- e^+$$  \hspace{1cm} (39)

and

$$\tau^- \rightarrow e^- e^- \mu^+,$$  \hspace{1cm} (40)

$$\tau^+ \rightarrow \mu^+ \mu^+ e^-$$  \hspace{1cm} (41)

and

$$\tau^+ \rightarrow e^+ e^+ \mu^-$$  \hspace{1cm} (42)

Ulrike Regner, University of Vienna
The Frobenius group $T_7$ as a symmetry in the lepton sector

Interactions

Diagrams of the new $\tau^-$ decays

1.a First $\tau^-$ decay via $\Psi_1^0$

1.b Second $\tau^-$ decay via $\Psi_1^0$

1.c First $\tau^-$ decay via $\Psi_2^0$

1.d Second $\tau^-$ decay via $\Psi_2^0$
Experimental mass limits

Experimental upper limit:

\[ B(\tau^+ \rightarrow \mu^+ \mu^+ e^-) \leq 2.3 \times 10^{-8} \]  \hspace{1cm} (43)

\[ B(\tau^+ \rightarrow \mu^+ \nu_\mu \bar{\nu}_\tau) = 0.1736 \]  \hspace{1cm} (44)

\[ B(\tau^+ \rightarrow \mu^+ \mu^+ e^-) = \frac{9m_\tau^2 m_\mu^2 (m_1^2 + m_2^2)^2}{m_1^4 m_2^4} B(\tau^+ \rightarrow \mu^+ \nu_\mu \bar{\nu}_\tau) \]  \hspace{1cm} (45)

Lower limit:

\[ \frac{m_1^2 m_2^2}{\sqrt{m_1^2 + m_2^2}} \geq 39 \text{ GeV} \]  \hspace{1cm} (46)
Principles of an experimental test of this theory

Principle: observation of decay products of $B - L$ gauge boson $Z'$

\[ Z' \rightarrow \psi_{1,2}^0 \bar{\psi}_{1,2}^0 \]  

(47)

Final states:

1. $\tau^+ \tau^- \mu^- \mu^+$
2. $\tau^+ \tau^+ \mu^- e^-$
3. $\tau^- \tau^- \mu^+ e^+$
4. $\tau^- \tau^+ e^+ e^-$
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

Only $\Phi_i$-terms (possible if supersymmetrised).

Note: supersymmetrisation not consistent with Yukawa terms

$(\bar{L}_L \Phi \ell_R$ and $-\bar{\nu}_R \bar{\Phi}^\dagger L_L)$

⇒ consider all quadratic and quartic terms

Quadratic terms:

$$
\sum_{i=1}^{3} \Phi_i^\dagger \Phi_i
$$

(48)
Quartic terms of the scalar potential of $\Phi$

Tensor product

$$(3 \otimes 3^*) \otimes (3 \otimes 3^*) = (1_1 \oplus 1_2 \oplus 1_3 \oplus 3 \oplus 3^*) \otimes (1_1 \oplus 1_2 \oplus 1_3 \oplus 3 \oplus 3^*) \quad (49)$$

Singlets:

1. $1_1 \otimes 1_1$
2. $1_2 \otimes 1_3 = 1_3 \otimes 1_2$
3. $3 \otimes 3^* = 3^* \otimes 3$
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

Quartic terms of the scalar potential of $\Phi$

$1_1 \otimes 1_1$ corresponds to

$$\left( \sum_{i=1}^{3} \Phi_i^\dagger \Phi_i \right)^2$$

(50)

→ sum of terms of the form

$$\Phi_i^\dagger \Phi_i \Phi_j^\dagger \Phi_j$$

(51)
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

Quartic terms of the scalar potential of $\Phi$

$1_2 \otimes 1_3$ corresponds to

$$(\Phi_1^\dagger \Phi_1 + \omega \Phi_2^\dagger \Phi_2 + \omega^2 \Phi_3^\dagger \Phi_3) \cdot (\Phi_1^\dagger \Phi_1 + \omega^2 \Phi_2^\dagger \Phi_2 + \omega \Phi_3^\dagger \Phi_3) \quad (52)$$

$\rightarrow$ sum of terms of the form

$$\Phi_i^\dagger \Phi_i \Phi_j^\dagger \Phi_j, \quad (53)$$

terms with $i = j$ have $+1$ as factor, $i \neq j$ have factor $-\frac{1}{2}$
The complete scalar potential of $\Phi$

$3 \otimes 3^*$ corresponds to

$$\Phi_i^\dagger \Phi_j \Phi_j^\dagger \Phi_i$$

with $i \neq j$

Potential:

$$V_{\Phi} = \mu^2 \sum_{i=1}^{3} \Phi_i^\dagger \Phi_i$$

$$+ (\lambda_1 + \lambda_2) \sum_{i=1}^{3} (\Phi_i^\dagger \Phi_i)^2$$

$$+ (\lambda_1 - \frac{\lambda_2}{2}) \sum_{i,j=1,i\neq j}^{3} \Phi_i^\dagger \Phi_i \Phi_j^\dagger \Phi_j$$

$$+ \lambda_3 \sum_{i,j=1,i\neq j}^{3} \Phi_i^\dagger \Phi_j \Phi_j^\dagger \Phi_i$$

(54)

(55)
Minimum of the scalar potential of $\Phi$

VEV: $\nu_1 = \nu_2 = \nu_3 = \nu$

Minimum of potential:

$$V_{\Phi, \text{min}} = 3\mu^2 \nu^2 + 3(3\lambda_1 + 2\lambda_3)\nu^4 \quad (56)$$

Minimization condition on $\nu$:

$$\nu = \sqrt{-\frac{\mu^2}{6\lambda_1 + 4\lambda_3}} \quad (57)$$
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

Constraints on the parameters

- $\mu^2$ real and negative (condition for spontaneous symmetry breaking)
- $\lambda_1$, $\lambda_2$ and $\lambda_3$ real (potential hermitian, each term individually hermitian)
- lower bound on each $\lambda_i$ (physical requirement that potential is bounded from below)
- lower bounds $\rightarrow$ expression under the square in (57) positive
- $\rightarrow \nu$ real and positive
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

Mass matrices

General formalism to find mass matrices for the charged and the neutral scalars

General potential (Grimus, Lavoura (2002)):

$$V = \sum_{i,j=1}^{n_H} \mu_{ij}^2 \phi_i^\dagger \phi_j + \sum_{i,j,k,l=1}^{n_H} \lambda_{ijkl} \left( \phi_i^\dagger \phi_j \right) \left( \phi_k^\dagger \phi_l \right)$$  \hspace{1cm} (58)

In our case: $n_H = 3$, nonvanishing coefficients:

- $\mu_{ii}^2 = \mu^2$
- $\lambda_{iiii} = (\lambda_1 + \lambda_2)$
- $\lambda_{iiik} = (\lambda_1 - \frac{\lambda_2}{2})$
- $\lambda_{ijji} = \lambda_3$
The Frobenius group $T_7$ as a symmetry in the lepton sector

Mass matrices

Mass matrix of the charged scalars $\phi_i^\pm$

\[ M^2_{+ij} = \mu^2_{ij} + \Lambda_{ij}, \] (59)

where

\[ \Lambda_{ij} = \sum_{k,l=1}^{n_H} \lambda_{ijkl} v_k^* v_l \] (60)

Here:

\[ M^2_{+ij} = \frac{\mu^2 \lambda_3}{3\lambda_1 + 2\lambda_3} \left( \begin{array}{ccc} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{array} \right) \] (61)
The scalar potential of $\Phi$

Mass matrices

Mass matrix of the neutral scalars $\phi_i^\pm$

$$M_0^2 = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}$$, \quad (62)$$

where

$$A_{ij} = \Re(\mu_{ij}^2 + \Lambda_{ij} + K_{ij}') + \Re(K_{ij})$$, \quad (63)$$

$$B_{ij} = \Re(\mu_{ij}^2 + \Lambda_{ij} + K_{ij}') - \Re(K_{ij})$$, \quad (64)$$

$$C_{ij} = -\Im(\mu_{ij}^2 + \Lambda_{ij} + K_{ij}') - \Im(K_{ij})$$ \quad (65)$$

and

$$K_{ik} = \sum_{j,l=1}^{n_H} \lambda_{ijkl} V_j V_l$$ \quad (66)$$

$$K_{il}' = \sum_{j,k=1}^{n_H} \lambda_{ijkl} V_j V_k^*$$ \quad (67)$$
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

Mass matrices

Mass matrix of the neutral scalars $\phi_i^\pm$ for our model

Here:

$C = 0$ as $\lambda_i$, $\mu^2$ and $v$ are real

$$A = \frac{\mu^2}{3\lambda_1 + 2\lambda_3} \begin{pmatrix}
-2\lambda_1 - 2\lambda_2 & -2\lambda_1 + \lambda_2 - 4\lambda_3 & -2\lambda_1 + \lambda_2 - 4\lambda_3 \\
-2\lambda_1 + \lambda_2 - 4\lambda_3 & -2\lambda_1 - 2\lambda_2 & -2\lambda_1 + \lambda_2 - 4\lambda_3 \\
-2\lambda_1 + \lambda_2 - 4\lambda_3 & -2\lambda_1 + \lambda_2 - 4\lambda_3 & -2\lambda_1 - 2\lambda_2
\end{pmatrix}$$

$B = 0$
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

Mass matrices

Eigenvalue equations

\[ M_+^2 a = m_a^2 a \quad (68) \]

and

\[ M_0^2 \begin{pmatrix} \mathcal{R}(b) \\ \mathcal{I}(b) \end{pmatrix} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix} \begin{pmatrix} \mathcal{R}(b) \\ \mathcal{I}(b) \end{pmatrix} = m_b^2 \begin{pmatrix} \mathcal{R}(b) \\ \mathcal{I}(b) \end{pmatrix} \quad (69) \]

Rewrite (69) as

\[ (\mu^2_{ij} + \Lambda_{ij} + K'_{ij}) b_j + K_{ij} b^* = m_b^2 b_i \quad (70) \]
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

The existence of massless Goldstone bosons

The existence of massless Goldstone bosons in this model

Potential (55) is invariant under $O(4)$:

$$\Phi_i = \frac{1}{\sqrt{2}} \left( \begin{array}{c} \Re(\Phi_i^+) + i\Im(\Phi_i^+) \\ \Re(\Phi_i^0) + i\Im(\Phi_i^0) \end{array} \right)$$

(71)

four real parameters

$$\Phi_i^\dagger \Phi_i = \frac{1}{2} (\Re(\Phi_i^+)^2 + \Im(\Phi_i^+)^2 + \Re(\Phi_i^0)^2 + \Im(\Phi_i^0)^2)$$

(72)

Invariant $\Rightarrow SO(4)$ sufficient, six parameters
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

The existence of massless Goldstone bosons

The existence of massless Goldstone bosons in this model

Additionally: Invariance under all phase transformations:

$$\Phi_i \to e^{i\alpha_i} \Phi_i,$$ \hspace{1cm} (73)

$\Rightarrow$ Invariance under three independent $U(1)$ transformations
One transformation is weak hypercharge, already contained in $SO(4)$ symmetry

Independent parameters (group generators): 8
Three symmetry generators spontaneously broken $\Rightarrow$ Goldstone theorem
$\Rightarrow$ number of massless Goldstone bosons: $8 - 3 = 5$
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

The existence of massless Goldstone bosons

Schematic visualization of a simplified potential

Simplifications:

- neutral part of the potential only
- $\Phi_3^0$ set to a fixed value (real but otherwise arbitrary)
- imaginary parts of $\Phi_1^0$ and $\Phi_2^0$ set to zero
- coefficients $\mu^2$ and $\lambda_i$ set to arbitrary real values
Pseudo Goldstone bosons

Goldstone bosons corresponding to longitudinal modes of the $W$ and $Z$ vector bosons:

$$a_W = \frac{1}{\sqrt{3}\nu} (\nu, \nu, \nu)^T \quad (74)$$

and

$$b_Z = \frac{i}{\sqrt{3}\nu} (\nu, \nu, \nu)^T \quad (75)$$

Masses are zero!
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

The existence of massless Goldstone bosons

Pseudo Goldstone bosons

General expressions for physical charged and neutral scalars:

$$ S^+_a = \sum_{i=1}^{n_H} a_i^* \varphi_i^+ $$  \hspace{1cm} (76)

and

$$ S^0_b = \sqrt{2} \sum_{i=1}^{n_H} \Re (b_i^* (\varphi_i^0))' , $$  \hspace{1cm} (77)

where $(\varphi_i^0)' = (\varphi_i^0) - \nu$ such that VEV of $(\varphi_i^0)'$ is zero.
Pseudo Goldstone bosons

In our case,

\[ S_{aw}^+ = \frac{1}{\sqrt{3}} \sum_{i=1}^{3} \Phi_i^+ \]  

(78)

and

\[ S_{bz}^0 = \frac{\sqrt{2}}{\sqrt{3}} \sum_{i=1}^{3} \Im(\Phi_i^0)' \]  

(79)

Two pseudo Goldstone bosons used to create the masses of the \( W \) and \( Z \) bosons
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

The existence of massless Goldstone bosons

Two additional massless Goldstone bosons in this model

$M_+^2$: no other Goldstone boson fields exist

$M_0^2$:

\[
M_0^2 = \begin{pmatrix}
A & 0 \\
0 & 0 \\
\end{pmatrix}
\]  

Choose the vectors

\[
b_1 = \frac{i}{\sqrt{2}v} (-v, v, 0)^T
\]

and

\[
b_2 = \frac{i}{\sqrt{6}v} (v, v, -2v)^T
\]
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

The existence of massless Goldstone bosons

Physical neutral and charged scalars

\[ S_{b_1}^0 = -\Im (\Phi_1^0)' + \Re (\Phi_2^0)' \]  
\[ S_{b_2}^0 = \frac{1}{\sqrt{3}} (\Im (\Phi_1^0)'+ \Re (\Phi_2^0)' - 2\Re (\Phi_3^0)') \]

Found two additional massless Goldstone bosons!

$A$ is in general non singular, eigenvalues are

\[ m_{a_1}^2 = (4\lambda_3 - 3\lambda_2) \frac{\mu^2}{3\lambda_1 + 2\lambda_3}, \]  
\[ m_{a_2}^2 = (4\lambda_3 - 3\lambda_2) \frac{\mu^2}{3\lambda_1 + 2\lambda_3}, \]  
\[ m_{a_3}^2 = -2(4\lambda_3 + 3\lambda_1) \frac{\mu^2}{3\lambda_1 + 2\lambda_3} \]
Summary of massless Goldstone bosons

Charged sector: one charged complex massless Goldstone boson, remaining two charged complex particles are massive Higgs bosons.

Neutral sector: three massless real Goldstone bosons and three massive real Higgs bosons.

\(a_W\) and \(b_Z\): used to create the masses of the \(W\) and the \(Z\) bosons

\(b_1\) and \(b_2\) remain massless particles that exist in this model

\(\Rightarrow\) forces with infinite range
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

The existence of massless Goldstone bosons

The existence of forces with infinite range?

Investigated experimentally: Mostepanenko, Sokolov (1993)

- measurements of the Casimir force
- gravitation experiments of both the Eotvos and the Cavendish type
- several other experiments

Upper limit for $\lambda_n$ is (for $n = 1$) $10^{-47}$ in

$$\lambda_n (2z)^2 \frac{1}{r} \left( \frac{r_0}{r} \right)^{n-1}$$

(\(z\)...number of protons in the atom, \(r_0 = 1\text{F} = 10^{-15}\text{m}\), \(r\)...distance, \(\lambda_n\)...dimensionless coupling constant)

$\Rightarrow$ existence of the predicted forces can be excluded!
The Frobenius group $T_7$ as a symmetry in the lepton sector

The scalar potential of $\Phi$

The existence of massless Goldstone bosons

The existence of forces with infinite range?

Relevant coupling constants $f_i$:

$\nu$ is almost exactly 100 GeV, $f_i$ fixed by the value of $\nu$ and charged lepton masses:

$$f_1 \sim 10^{-6} \quad (89)$$
$$f_2 \sim 10^{-4} \quad (90)$$
$$f_3 \sim 10^{-2} \quad (91)$$

$\Rightarrow$ coupling constants many orders of magnitude larger than values in the experimentally allowed region!

(only tree-level approximation)
Conclusions

$T_7$ looks promising at first...

- has a set of irreps that can accommodate the multiplets, i.e. the particle content
- Yukawa terms lead to suitable mass matrix for the charged leptons
- Yukawa terms lead to suitable Dirac and Majorana mass matrices for the neutrinos
- plausible assumptions (some of which are made in the context of $B - L$ gauging) lead to a suitable seesaw neutrino mass matrix
$T_7$ looks promising at first...

- both mass hierarchy scenarios can be realized
- most importantly: the tribimaximal neutrino mixing matrix can be explained!
- new decays are predicted
- experimental test is possible by observing the decay of the new $B - L$ gauge boson $Z'$
But then...

- necessary supersymmetrization is in contradiction with the Yukawa terms
- scalar potential of only $\Phi$ leads to two massless Goldstone bosons that remain new massless particles
- these massless bosonic particles correspond to forces with infinite range
- such forces are experimentally excluded if the couplings are of the order of magnitude of the couplings in this model

By these arguments, it is excluded that this model describes the physical reality. This $T_7$ symmetry model is therefore ruled out as explanation of the symmetry in the lepton sector.