On the Higgs triplet extension of the Standard Model

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Outline

• Introduction
• The Higgs triplet model
• Neutrino mass terms
• Cross sections
• Decay widths
• Conclusions
Introduction
• The Standard Model of particle physics (SM) has to be extended due to unsolved problems like
  • Quantum gravity
  • baryon asymmetry
  • dark matter
  • hierarchy problem
  • strong CP-problem
  • neutrino masses
  • etc.
Introduction: Extension of the SM

• How can the SM be extended?
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- Fundamental extensions:
  - Introduction of a larger gauge group than $\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$ like in Grand Unifying Theories
  - Assumption of extra dimensions like in string theories
  - etc.
Introduction: Extension of the SM

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  • etc.
• Particular extensions:
  • Augmentation of a single sector of the SM like the scalar sector, etc.
Introduction: Extended scalar sector

- The scalar sector can be extended by introducing additional Higgs multiplets such as in
  - Two Higgs doublet models (SM + 2\(\phi\))
  - Zee model (SM + 2\(\phi\) + \(\eta^+\))
  - Zee-Babu model (SM + \(\eta^+\) + \(k^{++}\))
  - Higgs triplet model (SM + \(\Phi = (\phi^{++}, \phi^+, \phi^0)\))
  - etc.
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• What is the motivation for extending the scalar sector?
• In the SM neutrinos stay massless due to the absence of right-handed neutrino fields
• But recent experiments have discovered the phenomenon of neutrino flavour oscillations, which is only possible if neutrinos have non-zero and different masses
• One possibility to introduce neutrino masses is the so-called type-II see-saw mechanism, in which a complex scalar triplet with $Y = 2$, the Higgs triplet, is added to the scalar sector
Introduction: Extended scalar sector

- The Lagrangian of the Yukawa sector is enhanced with a gauge invariant coupling $\mathcal{L}_\Delta$ between the Higgs triplet and the lepton doublets.
- $\mathcal{L}_\Delta$ automatically leads to a Majorana Mass term for neutrinos at tree level proportional to $v_T$ (the vacuum expectation value of the neutral component of the Higgs triplet).
The Higgs triplet model
The Higgs triplet model: The idea

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• It was shown that additional scalar singlets $S^+, S^{++}$ and a scalar triplet $\Phi = (\phi^{++}, \phi^+, \phi^0)$ permit Yukawa couplings, which allow lepton flavour violating transitions like $\mu \to e\gamma$ and $\mu \to 3e$

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- This idea of an additional a scalar triplet was also used by G.B Gelmini and M. Roncadelli in 1981 in order to introduce neutrino masses [2]

The Yukawa Lagrangian in the lepton sector is given by [3]

\[ \mathcal{L}_Y = \sum_{\alpha,\beta} \left\{ -c_{\alpha\beta} \bar{\ell}_{\alpha R} \phi^\dagger L_{\beta L} + \frac{1}{2} f_{\alpha\beta} L^T_{\alpha L} C^{-1} i\tau_2 \Delta L_{\beta L} \right\} + \text{H.c.} \]

- \( \alpha, \beta \) : flavour indices
- \( L_{\alpha L} = (\nu_{\alpha}, \ell_{\alpha L}) \) : left-handed lepton doublets
- \( \ell_{\alpha R} \) : right-handed lepton singlets
- \( \phi \) : Higgs doublet
- \( \Delta \) : 2\times2 representation of the Higgs triplet
- \( C \) : charge conjugation matrix
- \( \tau_2 \) : second pauli matrix
- \( c_{\alpha\beta}, f_{\alpha\beta} \) : coupling matrices, \( f \) symmetric, i.e. \( f_{\alpha\beta} = f_{\beta\alpha} \)

The Higgs triplet model: The multiplets

- The multiplets transform under $U \in SU(2)$ as

\[
L_{\alpha L} \rightarrow UL_{\alpha L}, \quad \ell_{\alpha R} \rightarrow \ell_{\alpha R}, \quad \phi \rightarrow U\phi, \quad \Delta \rightarrow U\Delta U^+ \]
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- The $U(1)$ transformation properties are determined by their hypercharges:

<table>
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<tr>
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<th>$L_{\alpha L}$</th>
<th>$\ell_{\alpha R}$</th>
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<td>$Y$</td>
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The Higgs triplet model: The $2 \times 2$ representation

- The relation between the triplet and the $2 \times 2$ representation is given by

$$\Delta = \Phi \cdot \vec{\tau} = \begin{pmatrix} H^+ & \sqrt{2}H^{++} \\ \sqrt{2}H^0 & -H^+ \end{pmatrix} \text{ with } \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}(H^0 + H^{++}) \\ \frac{1}{\sqrt{2}}(H^0 - H^{++}) \\ H^+ \end{pmatrix}$$
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\frac{1}{\sqrt{2}}(H^0 - H^{++}) \\
H^+
\end{pmatrix}
$$

- The charge eigenfields are given by

$$
H^{++} = \frac{1}{\sqrt{2}}(H_1 - iH_2), \quad H^+ = H_3, \quad H^0 = \frac{1}{\sqrt{2}}(H_1 + iH_2)
$$
The Higgs triplet model: VEVs

• The VEVs of the Higgs multiplets consistent with electric charge conservation are given by

\[
\langle \phi \rangle_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \quad \text{and} \quad \langle \Delta \rangle_0 = \begin{pmatrix} 0 & 0 \\ \nu_T & 0 \end{pmatrix}
\]
The Higgs triplet model: VEVs

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- We have set \( \langle H^0 \rangle_0 = \frac{\nu_T}{\sqrt{2}} \)

- We expect \( |\nu_T| \ll \nu \), since a larger triplet VEV would destroy the tree-level relation \( M_W = M_Z \cos(\theta_W) \) between the gauge boson masses and the Weinberg angle and precision measurements place a stringent bound on \( \nu_T \)[4]

The Higgs triplet model: The potential

- The most general Higgs potential involving $\phi$ and $\Delta$ is given by

$$V(\phi, \Delta) = a\phi^\dagger \phi + \frac{b}{2} \text{Tr} (\Delta \Delta^\dagger) + c(\phi^\dagger \phi)^2 + \frac{d}{4} (\text{Tr}(\Delta \Delta^\dagger))^2$$

$$+ \frac{e-h}{2} \phi^\dagger \phi \text{Tr}(\Delta \Delta^\dagger) + \frac{f}{4} \text{Tr}(\Delta^\dagger \Delta^\dagger) \text{Tr}(\Delta \Delta)$$

$$+ h\phi^\dagger \Delta^\dagger \Delta \phi + (t\phi^\dagger \Delta \tilde{\phi} + \text{H.c.})$$

with $\tilde{\phi} = i\tau_2 \phi^*$
• Under the assumption of lepton number conservation one has to assign lepton number -2 to the Higgs triplet and 0 to the Higgs doublet.
The Higgs triplet model: The potential

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- This lepton number is explicitly broken by the $t$-term ($t\phi^\dagger \Delta \tilde{\phi} + \text{H.c.}$) in the potential.
- All parameters of the potential are real, except $t$ which is complex in general, i.e. $t = |t| e^{i\omega}$. 
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• The doublet VEV \(v\) can be chosen real, by performing global U(1) transformation
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- All parameters of the potential are real, except \(t\) which is complex in general, i.e. \(t = |t|e^{i\omega}\).
- The doublet VEV \(\nu\) can be chosen real, by performing a global \(U(1)\) transformation.
- Because of the t-term we do not have a second global symmetry, the lepton number to make \(\nu_T\) real therefore we can write \(\nu_T = w e^{i\gamma}\) with \(w = |\nu_T|\).
The Higgs triplet model: The potential

- Following orders of magnitude for the parameters of the potential are assumed:

\[ a, b \sim v^2; \ c, d, e, f, h \sim 1; \ |t| \ll v \]
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  \[ a, b \sim v^2; \ c, d, e, f, h \sim 1; \ |t| \ll v \]

- The potential as function of the VEVs is given by

  \[
  V(\langle \phi \rangle_0, \langle \Delta \rangle_0) = \frac{1}{2} av^2 + \frac{1}{2} bw^2 + \frac{1}{4} cv^4 + \frac{1}{2} dw^4 \\
  + \frac{e-h}{4} v^2 w^2 + v^2 w |t| \cos(\omega + \gamma)
  \]

- It has to be minimized with respect to \( v, w \) and \( \gamma \) in order to obtain relations between parameters of the potential.
The Higgs triplet model: Minimum conditions

- Minimization with respect to $\gamma$, the phase of $v_T$, gives
  \[ \omega + \gamma = \pi \] or
  \[ v_T = -we^{-i\omega} \quad \text{and} \quad v_T t = -w|t| \]

- With this relation the other two minimum conditions are
  \[ a + cv^2 + \frac{e - h}{2}w^2 + 2|t|w = 0 \]
  \[ b + dw^2 + \frac{e-h}{2}v^2 + \frac{|t|}{w}v^2 = 0 \]

- We find the approximate solution
  \[ v^2 \approx \frac{a^2}{c} \quad \text{and} \quad w \approx |t|\frac{v^2}{b + (e-h)v^2/2} \]
• We see that $w \sim |t|$, the triplet VEV is of the order of the parameter $|t|$ in the potential
• The fine-tuning to get a small triplet is therefore simply given by $|t| \ll v$
• Alternatively, one could use $b \gg v^2$ to get a small triplet VEV
The Higgs triplet model: Mass terms

- Mass terms for charged leptons and neutrinos are induced by $\mathcal{L}_Y$ and the VEVs $\langle \phi \rangle_0, \langle \Delta \rangle_0$:

$$ - \left( \bar{\ell}_R \mathcal{M}_\ell \ell_L + \text{H.c.} \right) \quad \text{with} \quad \mathcal{M}_\ell = \frac{\nu}{\sqrt{2}} \left( c_{\alpha \beta} \right) $$

$$ \frac{1}{2} \nu_L^T C^{-1} \mathcal{M}_\nu \nu_L + \text{H.c.} \quad \text{with} \quad \mathcal{M}_\nu = \nu_T \left( f_{\alpha \beta} \right) $$
Neutrino mass terms
Neutrino mass terms: Dirac vs. Majorana neutrinos

- Majorana Fermions are their own anti-particles
- The equation for Majorana field is the same as for Dirac fields:
  \[
  (i\gamma^\mu \partial_\mu - m)\psi = 0
  \]
- The Majorana nature is hidden in the Majorana condition:
  \[
  \psi = \psi^C = C\gamma_0^T \psi^*
  \]
- Most of the SM extensions suggest that neutrinos have Majorana Nature
- In consequence of the smallness of the neutrino masses, it is difficult to distinguish between Dirac and Majorana neutrinos
- Neutrinoless $\beta\beta$-decay to be the only prospective road so far
Neutrino mass terms: Dirac mass term

- With two independent chiral 4-spinor fields $\nu_{L,R}$ one can construct a Dirac mass term by writing a Lorentz-invariant bilinear for Dirac fields [5]:
  \[ \bar{\nu}_R \mathcal{M} \nu_L + \text{H.c.} = \bar{\nu}' \hat{m} \nu' \]
- $\mathcal{M}$ is an arbitrary $n \times n$ mass matrix
- $\hat{m}$ is a diagonal and positive mass matrix with the bidiagonalization $U_R^\dagger \mathcal{M} U_L = \hat{m}$
- The physical Dirac fields are given by
  \[ \nu' = \nu'_L + \nu'_R \text{ with } \nu_{L,R} = U_{L,R} \nu'_{L,R} \]

Neutrino mass terms: Majorana mass term

- With only one chiral 4-spinor field $\nu_L$ a *Lorentz-invariant* bilinear can still be constructed with the help of the charge conjugation matrix $C$. This bilinear is the so called Majorana mass term [5]:

$$\frac{1}{2} \nu_L^T C^{-1} \mathcal{M} \nu_L + \text{H.c.} = -\frac{1}{2} \bar{\nu}' \hat{m} \nu'$$

- $\mathcal{M}$ is now a complex and symmetric $n \times n$ mass matrix
- $\hat{m}$ is a diagonal and positive mass matrix with the diagonalization $U_L^T \mathcal{M} U_L = \hat{m}$
- The physical Majorana fields are given by

$$\nu' = \nu'_L + (\nu'_L)^C \text{ with } \nu_L = U_L \nu'_L$$

- The Majorana mass term violates not only the individual lepton family numbers just as the Dirac mass term, but it also violates the total lepton number $L = \sum_{\alpha} L_{\alpha}$

Neutrino mass terms: The mixing matrix $U_{\text{PMNS}}$

- Situation in the lepton sector analogue to the quark sector:
  Flavour eigenfields ≠ mass eigenfields
- Neutrino mixing given by
  $$\nu_{\alpha L} = \sum_j U_{\alpha j} \nu_{jL}$$
- The lepton mixing matrix $U_{\text{PMNS}}$ (Pontecorvo-Maki-Nakagawa-Sakata-matrix) is given as
  $$U = U_{L}^{(\ell)^\dagger} U_{L}^{(\nu)}$$

where $U_{L}^{(\ell)^\dagger}$ and $U_{L}^{(\nu)}$ are the matrices which (bi)diagonalize the charged lepton/neutrino mass matrices.
Neutrino mass terms: The mixing matrix $U_{PMNS}$

- The lepton mixing matrix is usually parametrized as

$$U_{PMNS} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & -s_{12} & 0 \\ -s_{12} & c_{23} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\times \begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with $c_{ij} = \cos(\theta_{ij})$, $s_{ij} = \sin(\theta_{ij})$, $\delta$ is non-zero only if neutrino oscillations violate CP symmetry. $\alpha_1, \alpha_2$ are physically meaningful if neutrinos have Majorana nature.
Cross sections and decay widths
Cross sections and decay widths

- The Lagrangian $\mathcal{L}_Y$ permits lepton flavour violating processes $\alpha^- \beta^- \rightarrow \gamma^- \delta^-$ (for $\alpha, \beta, \gamma, \delta = e, \mu, \tau$)
Cross sections and decay widths

- The cross section was calculated as [6]
  \[ \sigma(\alpha^-\beta^- \to \gamma^-\delta^-) = \frac{|f_{\alpha\beta}|^2|f_{\gamma\delta}|^2}{16\pi(1 + \delta_{\gamma\delta})} \frac{s}{(s - m^2)^2 + m^2\Gamma^2} \]

- \( f \) Yukawa coupling matrix
- \( \delta_{\gamma\delta} \) Kronecker delta
- \( s = (p_1 + p_2)^2 \) Mandelstam variable
- \( m \) mass of \( H^-^- \)
- \( \Gamma \) total width of \( H^-^- \)

Cross sections and decay widths

- The gauge coupling

\[ \mathcal{L}_{\Delta gauge} = \frac{1}{2} \text{Tr}\{(D_\mu \Delta)^\dagger (D^\mu \Delta)\} \]

permits lepton number violating processes like \( e^- e^- \rightarrow W^- W^- \)
Cross sections and decay widths

- The cross section was calculated as
  \[
  \sigma(e^-e^- \rightarrow W^-W^-) = \frac{G_F |f_{ee}|^2 |\nu_T|^2}{4\pi} \frac{(s - 2m_W^2)^2 + 8m_W^4}{(s - m)^2 + m^2\Gamma^2} \sqrt{1 - \frac{m_W^2}{m^2}}
  \]

- $G_F$ Fermi constant
- $f$ Yukawa coupling matrix
- $\nu_T$ triplet VEV
- $m$ mass of $H^{--}$
- $\Gamma$ total width of $H^{--}$
- $m_W$ mass of the W-boson
Cross sections and decay widths

- Lets take a look at some decays of $H^{--}$
- $\Gamma(H^{--} \rightarrow \gamma^- \delta^-) = \frac{|f_{\gamma\delta}|^2}{4\pi(1+\delta_{\gamma\delta})} m$
- $\Gamma(H^{--} \rightarrow W^- W^-) = \frac{g^4|v_T|^2 (s-2m_w^2)^2+8m_w^4}{32\pi m} \frac{m_w^4}{m^2} \sqrt{1 - \frac{m_w^2}{m^2}}$
Cross sections and decay widths

\[ \Gamma(H^{--} \to H^- W^-) = \frac{g^2}{16\pi m^3 m_w^2} \left[ \lambda(m^2, m_-^2, m_W^2) \right]^3 \]

- \( m \) mass of \( H^{--} \)
- \( m_- \) mass of \( H^- \)
- \( m_W \) mass of the W-boson
- \( g \) coupling constant
- \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) \)
Cross sections and decay widths

- The 3-body decay $H^{-} \rightarrow H^{-} W^{-} Z^{0}$ has four contributions:
Cross sections and decay widths

- The squared amplitude $|\mathcal{M}|^2$ of this process consists of over a hundred terms.
- Most of the terms contain $H^-, H^{--}$ or $W^-$ propagators.
- The 3-body phase-space integral over the rational functions was not solvable analytically.
- Therefore $\Gamma(H^{--} \to H^- W^- Z^0)$ was solved numerically with the FORTRAN program RAMBOC, based on "RAMBO" (random momenta beautifully organized)[7].
- "RAMBO" is based on the Monte Carlo algorithm. The integration over phase space is replaced by a number of random choices over the integration variable.

Cross sections and decay widths
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Conclusions
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• The additional term in the Yukawa Lagrangian $\mathcal{L}_Y$ induced a lepton number violating Majorana mass term for neutrinos at tree level, which was found as $\frac{1}{2} \nu_L^T C^{-1} \mathcal{M}_\nu \nu_L + \text{H.c.}$

• The mass matrix was given by $\mathcal{M}_\nu = \nu_T (f_{\alpha\beta})$

• The triplet VEV had to be very small, i.e. $|\nu_T| \ll \nu$, which was achieved by the fine-tuning $|t| \ll \nu$; $t$ was the parameter of the lepton number violating term in the potential $(t \phi^\dagger \Delta \phi + \text{H.c.})$

• Neutrinos can be Dirac or Majorana fermions, most SM extension suggest Majorana nature of the neutrino

• $\mathcal{L}_Y$ permits interesting lepton flavour and lepton number violating processes like $\alpha^- \beta^- \rightarrow \gamma^- \delta^-$, $e^- e^- \rightarrow W^- W^-$, $H^- \rightarrow H^- W^-$, $H^- \rightarrow H^- W^- Z^0$
Thank you for your attention!